

# On the ac conductivity of slightly disordered one-dimensional conductors

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The Austin-Mott formula for the frequency dependence of the conductivity is derived for the case of one-dimensional conductors containing disorder. It is shown under what circumstances the “pair approximation” is valid in these systems. The conditions, for which the two states making up a pair are statistically independent, are considered.

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The calculation of the conductivity for localized electrons (say, in semiconductors) is greatly simplified by model representations.<sup>1</sup> An electron is localized at separate centers, and far from the centers its wave function decays exponentially with some atomic constant  $a$ . The overlap is ignored—at low concentrations of centers. At low temperatures only the transitions between pairs of nearest centers are important, since the probability of finding two levels  $(\mu, \nu)$  with nearby energies at small spatial separations is generally small. If the transitions between states are due to phonons, the corresponding relaxation time is<sup>2</sup>

$$1/\tau_{\mu\nu} = \nu_{ph} \exp(-2z/a). \quad (1)$$

At zero temperature and for  $\omega \rightarrow 0$ , the absorption of energy from the external field is due to transitions between the two quantum states formed by the tunneling splitting of the levels of two “identical” wells<sup>3</sup>:

$$\hbar\omega = I_0 \exp(-r/a). \quad (2)$$

This expression implies the well-known Mott law for the phononless conductivity,  $\sigma(\omega) \propto \omega^2 \ln^2 I_0/\omega$  (in the 1D case).

According to Refs. 3 and 4, a one-dimensional system containing disorder has only localized states even in the limit  $k_F l \gg 1$ , which would otherwise correspond to a regime of metallic conductivity. As an initial approximation here one takes the rapidly oscillating band wave functions. The density of states  $\nu(E_F) = (\hbar\pi v_F)^{-1}$  is not small, and the energy scale is determined by the reciprocal  $1/\tau$  of the characteristic time for (back)scattering by defects (the range is  $l = v_F \tau$ ).

Our goal here is to study the applicability of the phenomenological representations (1) and (2) to this case. In particular, we shall obtain the Austin-Mott formula for the conductivity.<sup>2</sup>

The spectrum of the Schrödinger equation in a 1D system containing disorder consists of localized states with “unit” probability (see Ref. 5). The (Lyapunov) growth exponent of the amplitude  $R(x)$  of the wave function  $\psi(x) = R(x)\sin\phi(x)$  is a self-averaging quantity which is given by<sup>6</sup>

$$\lim_{z \rightarrow \infty} \frac{1}{z} \ln \frac{R(z)}{R(0)} = \frac{1}{2l} . \quad (3)$$

Consequently, we have  $a = 2l$  in (1).

The phononless contribution to the conductivity  $\sigma(\omega)$  is determined by the average of the square of the dipole-moment matrix element  $x_{\mu\nu}$  between two states  $(\mu, \nu)$ :

$$\sigma(\omega) \propto \left\langle \sum_{\mu\nu} \delta(E - \epsilon_\mu) \delta(E + \omega - \epsilon_\nu) x_{\mu\nu}^2 \right\rangle .$$

The latter is expressed in terms of a density-density correlator of the two eigenfunctions:

$$F_\omega^1(z) = \left\langle \sum_{\mu\nu} \delta(E - \epsilon_\mu) \delta(E + \omega - \epsilon_\nu) \psi_\mu(0) \psi_\nu(0) \psi_\mu(z) \psi_\nu(z) \right\rangle . \quad (4)$$

We are interested in the case  $\omega\tau \ll 1$ . If all the localized wave functions were independent, then, according to (3), the function  $F_\omega^{(1)}(z)$  would have a peak at  $z \sim l$  and be small for  $z \gg l$ . A calculation (which we shall not reproduce here) shows that this correlator has another peak as well:

$$F_\omega^{(1)}(z) = -\nu^2 (E_F) (l/4\pi z_0)^{1/2} \exp[-(z - z_0)^2/4lz_0] . \quad (5)$$

at

$$z_0 = 2l \ln(8/\omega\tau) . \quad (6)$$

The properties of  $F_\omega^{(1)}(z)$  should be discussed in conjunction with the properties of the correlator of the densities of states at two different points<sup>7</sup>:

$$F_\omega(z) = \left\langle \sum_{\mu\nu} \delta(E - \epsilon_\mu) \delta(E + \omega - \epsilon_\nu) \psi_\mu^2(0) \psi_\nu^2(z) \right\rangle . \quad (7)$$

The behavior of the correlators  $F_\omega(z)$  and  $F_\omega^{(1)}(z)$  is shown in Fig. 1 (for  $\omega\tau \ll 1$ ) by the solid and dashed curves, respectively. For  $z \rightarrow \infty$  we have  $F_\omega(z) = \nu^2(E_F)$ , which corresponds to statistically independent states at large distances. For  $z < z_0$  a "dip" appears, demonstrating a "repulsion" of the levels. However, there is no "repulsion" at small distances  $z \sim l$ , where the two functions coincide (the hatched region in Fig. 1 corre-

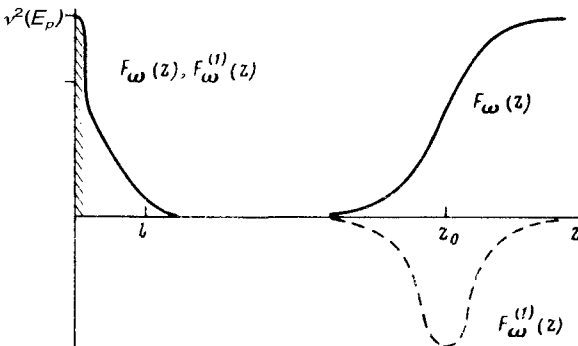


FIG. 1.

sponds to the closest distances  $z \sim k_F^{-1}$ ). The figure indicates the presence of a strong correlation of two nearby levels for  $z \lesssim z_0$ .

Molcanov<sup>8</sup> has proved that for a very long chain ( $L \rightarrow \infty$ ), "all" of the energy levels are statistically independent. We have evaluated the probability  $W_N(L, \omega)$  that  $N$  levels appear in the energy interval  $\omega$  for a chain of finite length  $L$  ( $L < [\omega v(E_F)]^{-1}$ ):

$$W_N(L, \omega) = (N!)^{-1} [\omega v(E_F) (L - N z_0)]^N. \quad (8)$$

This expression contains the so-called "exceptional volume," which indicates once again that the levels do not approach each other closer than  $z_0$ . At these distances the two levels cease to be independent and form a correlated pair analogous to the two coherent states which arise on account of tunneling in the two identical potential wells.<sup>3,9</sup> [Ternary correlations are already missing in (8)]. The coincidence of the two correlators for  $z \sim l$  indicates the possibility of a limiting transition  $\epsilon_\mu - \epsilon_\nu \rightarrow 0$ , i.e., the phases of these states are nearly the same. Analysis of the behavior of the phase difference from  $z \sim l$  to  $z \sim z_0$  shows that if the two wave functions are correlated for a given state of disorder, this corresponds to the rare case in which the phase advance at the distance  $z_0$  is equal to  $\pi$ . The large value of  $z_0$  [Eq. (6)] permits interpretation of the shape and width of the distribution of  $F_\omega^{(1)}(z)$  in terms of Gaussian fluctuations of the defect concentration. These results establish the limits of applicability of the model representations of localized and independent states.

The frequency dependence of the conductivity for  $T \neq 0$  is due to the Debye loss,<sup>1,10</sup> i.e., to the relaxation ( $1/\tau_{\mu\nu}$ ) due to phonons of the populations of two neighboring (close in both energy and space) levels. The contribution to the dissipation from an individual pair ( $\mu, \nu$ ), as we know,<sup>10,11</sup> is proportional to the factor

$$\frac{\omega^2}{\tau_{\mu\nu}} \left[ \omega^2 + \frac{1}{\tau_{\mu\nu}^2} \right]^{-1}; \quad (9)$$

it follows that the important pairs are those for which  $1/\tau_{\mu\nu} \sim \omega$ . If the one-dimensional filaments make up a crystal, the transitions between states at low temperatures are governed by three-dimensional acoustic phonons. An estimate of the transition matrix elements by the method of Ref. 12 gives

$$\frac{1}{\tau_{\mu\nu}} = \frac{\alpha g_{ph}^2}{n_\perp l^2} T (\tau T)^2 \frac{v_F}{s} \frac{\Delta \epsilon}{T} \text{cth} \frac{\Delta \epsilon}{T} \exp\left(-\frac{z - z_0}{l}\right), \quad (10)$$

where  $\Delta \epsilon = \epsilon_\mu - \epsilon_\nu$ ,  $s$  is the speed of sound,  $n_\perp$  is the density of filaments,  $g_{ph}^2$  is the dimensionless electron-phonon interaction constant, and  $\alpha$  is a numerical factor. In expression (10) it is explicitly taken into account that  $z > z_0$  in (6). With this expression we obtain, by averaging (9), the Austin-Mott formula:

$$\sigma(\omega) = \frac{\pi^3}{4} e^2 v^2 (E_F) n_\perp l^3 \omega T \ln^2 \left( \frac{T}{\hbar \omega} \frac{v_F}{s n_\perp l^2} \right). \quad (11)$$

At high frequencies there is a plateau region, for which the following microscopic expression has been found<sup>12,13</sup>:

$$\sigma = g_{ph}^2 \frac{\pi^2}{45} \frac{e^2}{\hbar v_F} \frac{1}{\tau} \left( \frac{v_F}{s} \right) \left[ \frac{2 T \tau}{\hbar} \ln \left( \frac{2 T \tau}{\hbar} \right) \right]^4. \quad (12)$$

In (10) and (11),  $z - z_0$  is determined by the condition  $\omega \sim 1/\tau_{\mu\nu}$ . It is seen that (11) joins with (12) (i.e.,  $z \approx z_0$ ) at frequencies

$$\omega \sim \frac{\alpha g_{ph}^2}{n_{\perp} l^2} \frac{T}{\hbar} \left( \frac{T \tau}{\hbar} \right)^2 \frac{v_F}{s}.$$

In conclusion, we note that in  $1D$  conductors the long-range Coulomb interaction is screened: At distances shorter than  $l$  the screening is due to the displacement of the charges on neighboring filaments, while at larger distances the Coulomb term is simply small on account of the anomalously large dielectric constant (the localization effect<sup>4</sup>). The Coulomb effects proposed by Éfros and Shklovskii<sup>14,15</sup> are therefore small, in contrast to the case of semiconductors. The short-range (screened) interactions, although altering the coefficient in (11), do not otherwise introduce any new frequency or temperature dependence in the conductivity.

<sup>1</sup>N. F. Mott and E. A. Davis, *Electronic Processes in non-Crystalline Materials*, Clarendon Press, Oxford, 1971.

<sup>2</sup>I. G. Austin and N. F. Mott, *Adv. Phys.* **18**, 41 (1969).

<sup>3</sup>N. F. Mott and W. D. Twose, *Adv. Phys.* **10**, 107 (1961).

<sup>4</sup>V. L. Berezinskii, *Zh. Eksp. Teor. Fiz.* **65**, 1251 (1973) [*Sov. Phys. JETP* **38**, 620 (1974)].

<sup>5</sup>I. M. Lifshitz, S. A. Gredeskul, and L. A. Pastur, *Vvedenie v Teoriyu Neuporyadochennykh Sistem* [Introduction to the Theory of Disordered Systems], Nauka, Moscow, 1982.

<sup>6</sup>R. Borland, *Proc. R. Soc. London Ser. A* **274**, 809 (1963).

<sup>7</sup>L. P. Gor'kov, O. N. Dorokhov, and F. V. Prigara, *Zh. Eksp. Teor. Fiz.* **84**, 1440 (1983) [*Sov. Phys. JETP* **57** (1983)].

<sup>8</sup>S. A. Molcanov, *Commun. Math. Phys.* **78**, 429 (1981).

<sup>9</sup>I. M. Lifshitz, *Usp. Fiz. Nauk* **83**, 617 (1964) [*Sov. Phys. Usp.* **7**, 549 (1965)].

<sup>10</sup>M. Pollak and T. H. Geballe, *Phys. Rev.* **122**, 1742 (1961).

<sup>11</sup>H. Böttger and V. V. Bryksin, *Phys. Status Solidi B* **78**, 9, 415 (1976).

<sup>12</sup>V. L. Berezinskii, L. P. Gor'kov, and O. N. Dorokhov, *Solid State Commun.* **37**, 61 (1981).

<sup>13</sup>O. N. Dorokhov, S. I. Matveenko, and F. V. Prigara, *Fiz. Nizk. Temp.* **7**, 738 (1981) [*Sov. J. Low. Temp. Phys.* **7**, 360 (1981)].

<sup>14</sup>A. L. Éfros, *Philos. Mag. Ser. B* **43**, 829 (1981).

<sup>15</sup>B. I. Shklovskii and A. L. Éfros, *Zh. Eksp. Teor. Fiz.* **81**, 406 (1981) [*Sov. Phys. JETP* **54**, 218 (1981)].

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