

A mechanism for the transition to chaos in the system of an electron beam and an electromagnetic wave

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A mechanism observed for the transition to chaos in a multimode system of an electron beam and an electromagnetic wave exhibits features of the Landau picture of the transition to turbulence, and features of the Feigenbaum period-doubling bifurcation mechanism which is characteristic of few-mode systems.

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The possibility of describing chaos in many-mode systems by means of few-mode models which demonstrate a chaotic behavior has recently been the object of active research.¹⁻⁴

In this letter we report a study of the mechanism for the transition to many-mode chaos in a system consisting of a length of an electrodynamic transmission line penetrated by an electron beam.

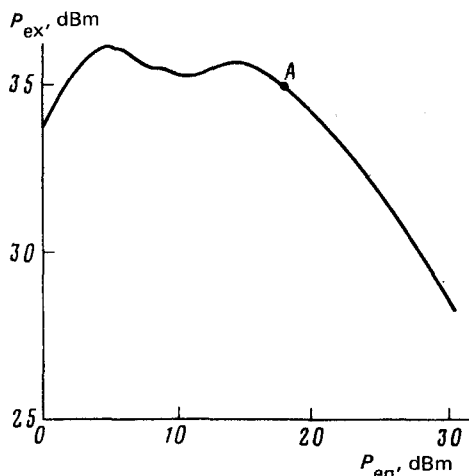


FIG. 1. Signal gain at the frequency of the first mode to be excited in an open system.

Some of the wave energy is returned from the exit end of the line to its entrance end to create the positive feedback required for oscillation in the system. The fraction of the wave energy delivered to the entrance end is described by the parameter $\gamma = 10 \log (P_{\text{en}}/P_{\text{ex}})$, where P_{en} and P_{ex} are the power levels at the entrance and the exit, respectively. Autostochastic oscillations are observed in the system when the beam current J , the accelerating voltage U , and the parameter γ satisfy certain relations.⁵ In this system it is possible to change the transmission band Δf and thus study regimes with various numbers of excited modes, from one to several tens. The distance between the natural modes is $\Omega \sim 1/\tau$, where τ is the time required for the wave to propagate along the line. The number of modes in which oscillations can be excited is $\beta \sim \Delta f \tau$.

The transition to chaos with increasing γ and at fixed values of the beam current and the accelerating voltage was studied for β from 3 to 15. The accuracy of the determination of γ was 0.05 dB. Under these conditions there is typically a transition to chaos by the mechanism for which the spectral evolution is shown in Fig. 2 for the case $\beta \sim 10$.

Oscillation occurs at $\gamma \sim -32$ dB at one of the frequencies, despite the fact that the phase balance conditions hold for all the frequencies in the transmission band. This single-frequency oscillation occurs as γ is raised to -17.1 dB, at which point other modes are excited, and the spectrum acquires components separated by a characteristic distance $\sim \Omega$ (Figs. 2b and 2c). The value $\gamma = -17.1$ dB corresponds to point A in Fig. 1, on the descending branch of the characteristic. In this region the efficiency of the signal gain at the frequency of the first mode to be excited falls off with increasing input power, causing a pumping of energy into other modes, previously suppressed. Analysis of the envelope of the signal shows that the excited modes are uncorrelated. With increasing γ , the process evolves continuously until γ reaches -15.6 dB. Up to this point, the transition to chaos is in complete agreement with Landau's picture of

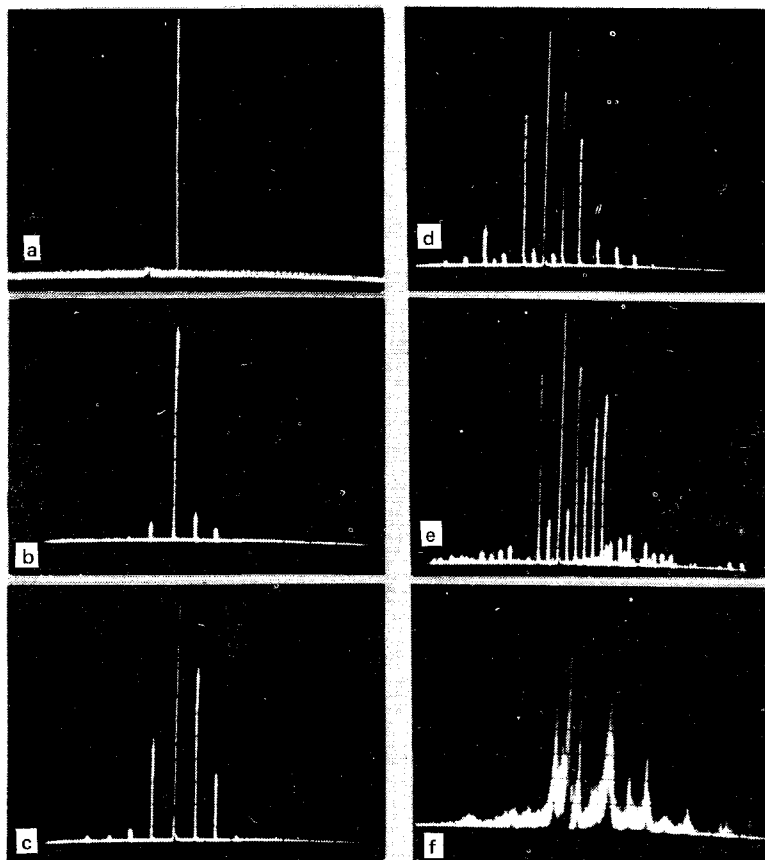


FIG. 2. Evolution of the signal spectrum.

the transition to turbulence.⁶ According to that picture, a monotonic change in some parameter first gives rise to a periodic motion; this motion eventually becomes unstable, and another oscillatory motion arises, with a frequency different from that of the first. As the parameter is changed further, the number of excited modes increases.

At $\gamma = -15.6$ dB, the nonlinear interaction of previously uncorrelated modes with equidistant frequencies gives rise to a synchronization and to a common periodic process with a period τ in the system (Fig. 3a). The process remains stable up to $\gamma = -13.1$ dB, beyond which point the period of the envelope of the process doubles (Fig. 3b), and spectral components appear with frequencies $f_i \pm \Omega/2$, (Fig. 1d), where f_i are the frequencies of the natural modes. Later, at $\gamma = -12.2$ dB, a new period doubling of the envelope occurs (Fig. 3c), and components with frequencies $f_i \pm \Omega/4$ appear in the spectrum. This stage of the evolution of the spectrum consists of two bifurcations of the period doubling of the envelope of the process and is similar to the beginning of the Feigenbaum chain of period doubling bifurcations.⁷

A signal with a continuous spectrum begins to be generated in the system at

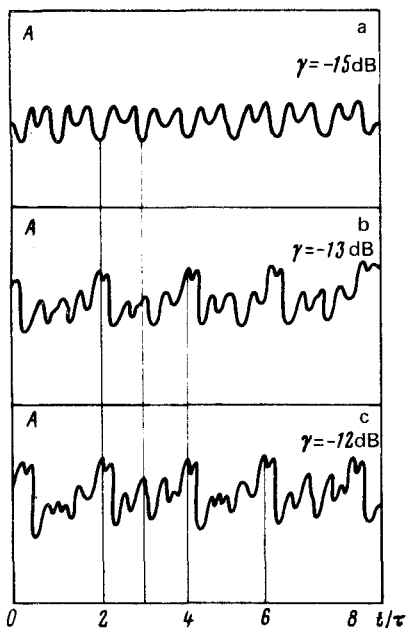


FIG. 3. Evolution of the envelope of the signal.

$\gamma = -11.2$ dB. A detailed analysis of the spectral evolution over the interval $-12.1 < \gamma < -11.2$ dB, carried out with a spectrum analyzer with a dynamic range of 60 dB and a frequency resolution $\sim 5 \times 10^{-3} \Omega$, revealed no period-doubling bifurcations beyond the second. The extent to which the signal exceeds the intrinsic noise level of the spectrum analyzer at the frequencies $f_i \pm \Omega/2$ and $f_i \pm \Omega/4$ is ~ 45 and ~ 25 dB, respectively. The transition to the continuous spectrum begins with a conversion of the components $f_i \pm \Omega/2$ and $f_i \pm \Omega/4$, and then of the natural modes to noise; then the noise background spreads out over the entire spectrum. With a further increase in γ the spectrum becomes slightly less choppy; the peaks remain in the region of the natural modes (Fig. 2f).

In conclusion we note that the statistics of the signal in the regimes with an extended continuous spectrum is nearly Gaussian.

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