Deep inelastic electron scattering by a polarized target in quantum chromodynamics

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Evolution equations are constructed for the nonsinglet part of the structure functions $g_1(x)$ and $g_2(x)$ in quantum chromodynamics. The anomalous dimensionalities of the corresponding twist-3 operators are calculated.

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1. The cross sections of several hard inclusive processes can be written[1] in the parton model in terms of the number densities of partons in a hadron, $D_h^i(x)$, and of hadrons in a parton, $\widetilde{D}_i^h(x)$, where x is the relative energy of the constituent particles in a system with an infinite momentum. This approach remains valid in the main logarithmic approximation ($g^2 \ln(q^2/m^2) \sim 1$, $g^2 \ll 1$) in the renormalizable models of quantum field theory[2] if the parameter of the ultraviolet cutoff in the transverse components of the parton momentum, $k_\perp$, is, in order of magnitude, $\Lambda \sim \sqrt{-q^2}$.

The following balance equations[2] hold for the parton densities with a change in Λ:

$$\frac{\partial D_j^i(x)}{\partial \ln \Lambda} = - W_i D_h^i(x) + \sum_{i'} \int_x^1 dx' W_{i' \to i}(x', x) D_j^{i'}(x'), \quad W_i = \sum_{j'} \int_0^x dx' W_{i \to j'}(x, x'). \tag{1}$$

The first term on the right describes the decrease in the number of partons of species i due to their decay into other partons, and the other terms describe the increase in the number of partons of species i due to the decay of other partons. The kernels of integral equations (1) are calculated in the order g_Λ^2 from simple single-loop eigenenergy diagrams. These kernels have been derived for the pseudoscalar model,[2] for quantum electrodynamics,[2] and for quantum chromodynamics.[3] Equations (1) are usually called "evolution equations."

In this letter we show that in order to describe the structure functions $g_1(x)$ and $g_2(x)$ for the deep inelastic scattering of electrons by a polarized nucleon in the main logarithmic approximation we need to introduce, in addition to $D_j^i(x)$, some auxiliary functions $Y(x_1, x_2)$ which correspond to the matrix elements of certain operators calculated between hadron states for which the number of gluons differs by one. The functions $Y(x_1, x_2)$ are analogous to the density matrix of the number of particles, and they satisfy evolution equations of type (1).

2. The degree of polarization of an initial nucleon with momentum p is characterized by the auxiliary vector $s^\sigma = u(p)\gamma^\sigma\gamma_5 u(p)$, $\bar{u}(p)u(p) = 2m$, and as a result an antisymmetric increment appears in the imaginary part $[W_{\mu\nu}(p, q)]$ of the amplitude $(T_{\mu\nu})$ for the zero-angle Compton effect[4] $(t = 0)$:

$$\frac{1}{\pi} W^a_{\mu\nu} = \frac{1}{2\pi}(W_{\mu\nu} - W_{\nu\mu}) = \frac{i}{pq}\epsilon_{\mu\nu\lambda\sigma} q^\lambda [g_1(Q^2,x)s^\sigma + g_2(Q^2,x) s_\perp^\sigma],$$

$$s_\perp^\sigma = s^\sigma - (sq)/(pq)\, p^\sigma, \quad Q^2 = -q^2, x = Q^2/2(pq). \tag{2}$$

In the operator expansion of the main logarithmic approximation the following equations hold for $g_{1,2}(Q^2, x)$:

$$\frac{(q's)}{(pq')} g_1(x) = -(1/2\pi)\mathrm{Im}\sum_{n=0}^{\infty} \omega^n (1 + (-1)^n \sum_i l_i^2 < p|R_{1,i}^{\overbrace{\cdots}^{n+1}}|p>, \quad \omega = 2pq/Q^2,$$

$$s_\perp^\sigma (g_1(x) + g_2(x)) = -\frac{1}{2\pi}\mathrm{Im}\sum_{n=0}^{\infty}(1 + (-1)^n)\omega^n \sum_i l_i^2 < p|R_{1i}^{\sigma\overbrace{\cdots}^{n}} + \frac{2n}{n+1} R_{2i}^{\sigma\overbrace{\cdots}^{n}}|p>, \tag{3}$$

where l_i is the charge of a quark of species i, expressed in units of the electron charge. The operators $R_{1i}^{\overbrace{\cdots}^{n+1}}$, $R_{1i}^{\sigma \overbrace{\cdots}^{n}}$, $R_{2i}^{\sigma \overbrace{\cdots}^{n}}$ are certain components of the tensors $R_{1i}^{\sigma\mu_1\cdots\mu_n}$ and $R_{2i}^{\sigma\mu_1\cdots\mu_n}$, which correspond to constituent operators with twist 2 and twist 3, respectively:

$$R_{1i}^{\overbrace{\cdots}^{n+1}} = (pq')^{-n-1} q'_{\mu_1}\cdots q'_{\mu_{n+1}} R_{1i}^{\mu_1\cdots\mu_{n+1}},$$

$$R_{ri}^{\sigma\overbrace{\cdots}^{n}} = (pq')^{-n} q'_{\mu_1}\cdots q'_{\mu_n} R_{ri}^{\sigma\mu_1\cdots\mu_n}, \quad q' = q + xp, \quad q'^2 = 0. \tag{4}$$

These operators are given by the following expressions (for simplicity, we omit the subscript "i" from the operators and from the quark fields ψ)[4,5]:

$$R_1^{\sigma\mu_1\cdots\mu_n} = i^n \underset{\sigma\mu_1\cdots\mu_n}{S} \bar\psi\gamma_5\gamma^\sigma D^{\mu_1}\cdots D^{\mu_n}\psi - \text{traces},$$

$$R_2^{\sigma\mu_1\cdots\mu_n} = i^n \underset{\mu_1\cdots\mu_n}{S}\ \underset{\mu_1\sigma}{A}\ \underset{\mu_1\cdots\mu_n}{S} \bar\psi\gamma_5\gamma^\sigma D^{\mu_1}\cdots D^{\mu_n}\psi - \text{traces}, \tag{5}$$

where $\underset{\sigma\mu_1\cdots\mu_n}{S}$ and $\underset{\sigma\mu_1}{A}$ represent symmetrization and antisymmetrization in the corresponding Lorentz indices, and the D^μ constitute the covariant derivative.

It is assumed in (3) that the operators are normalized at $\Lambda = \sqrt{Q^2}$. Upon a change in the normalization point, the operator R_2 may mix with the following twist-3 operator[4]:

$$R_3^{\sigma\mu_1\cdots\mu_n} = i^{n-1}\, m \underset{\mu_1\cdots\mu_n}{S}\ \underset{\mu_1\sigma}{A} \bar\psi\gamma_5\gamma^\sigma\gamma^{\mu_1} D^{\mu_2}\cdots D^{\mu_n}\psi - \text{traces}, \tag{6}$$

where m is the bare quark mass.

The operators $R_2^{\sigma\overbrace{\cdots}^{n}}$ and $R_3^{\sigma\overbrace{\cdots}^{n}}$ do not exhaust the list of operators between which mixing is possible during renormalizations. There are two families of twist-3 operators which must be taken into account in calculating the matrix of anomalous dimensionalities:

$$R_{4l}^{\sigma\mu_1\ldots\mu_n} \cong i^{n-3} \underset{\mu_1\ldots\mu_n}{S} \bar{\psi} D^{\mu_1}\ldots D^{\mu_l}\, g\tilde{G}^{\sigma\mu_{l+1}} D^{\mu_{l+2}}\ldots D^{\mu_{n-1}}\gamma_5\gamma^{\mu_n}\psi,$$

$$R_{5l}^{\sigma\mu_1\ldots\mu_n} = i^{n-2} \underset{\mu_1\ldots\mu_n}{S} \bar{\psi} D^{\mu_1}\ldots D^{\mu_l}\, gG^{\sigma\mu_{l+1}} D^{\mu_{l+2}}\ldots D^{\mu_{n-1}}\gamma^{\mu_n}\psi, \tag{7}$$

$l = 0, \ldots n-2,$

where g is the bare charge, $G^{\sigma\rho} = \frac{-i}{g}[D^\sigma, D^\rho]$ is the stress tensor of the gluon field, and $\tilde{G}^{\sigma\rho} = \frac{1}{2}\varepsilon^{\sigma\rho\mu\nu}G_{\mu\nu}$ is a dual tensor. This fact was not noted in Ref. 4, and the results derived there are accordingly incorrect. The existence of the set of operators R_{4l}, R_{5l} was recognized independently by Shuryak and Vaĭnsteĭn,[5] and the anomalous dimensionalities for the moment $n = 2$ were calculated by a method which incorporated the mixing of these operators with R_2, R_3. In the present letter we construct the anomalous-dimensionality matrix for arbitrary n. We consider only the case which is nonsinglet in the flavor quantum numbers in the t channel, where there is no purely gluon intermediate state.

3. An axial gauge for the vector potential $A_\mu (q'_\mu A^\mu = 0)$ is convenient for the calculation,[2] since in this case we need consider only the diagrams having two and three particles in the t channel.

We write the matrix elements of the operators R_r between hadron states as integrals over the parton distributions:

$$\langle p|R_1^{\overbrace{\cdots}^{n+1}}|p\rangle = \frac{sq'}{pq'}\int d\beta\, \beta^{n+1} E(\beta) = \frac{sq'}{pq'}E_n,$$

$$\langle p|R_1^{\sigma\overbrace{\cdots}^{n}}|p\rangle = \frac{S_\perp^\sigma}{n+1}\int d\beta\beta^{n-1} E(\beta) = \frac{S_\perp^\sigma}{n+1}E_n,$$

$$\langle p|R_1^{\sigma\overbrace{\cdots}^{n}} + \frac{2n}{n+1}R_2^{\sigma\overbrace{\cdots}^{n}}|p\rangle = S_\perp^\sigma \int d\beta\beta^{n-1}A(\beta) = S_\perp^\sigma A_n,$$

$$\langle p|R_3^{\sigma\overbrace{\cdots}^{n}}|p\rangle = S_\perp^\sigma \int d\beta\beta^{n-1}C(\beta) = S_\perp^\sigma C_n, \tag{8}$$

$$\langle p|R_{4l}^{\sigma\overbrace{\cdots}^{n}} - R_{4n-l}^{\sigma\overbrace{\cdots}^{n}}|p\rangle = S_\perp^\sigma\int d\beta_1 d\beta_2 \beta_1^{l-1}\beta_2^{n-l-1}(Y(\beta_1,\beta_2) - Y(\beta_2,\beta_1)) = -S_\perp^\sigma(Y_n^l - Y_n^{n-l}),$$

$$\langle p|R_{5l}^{\sigma\overbrace{\cdots}^{n}} - R_{5n-l}^{\sigma\overbrace{\cdots}^{n}}|p\rangle S_\perp^\sigma\int d\beta_1 d\beta_2 \beta_1^{l-1}\beta_2^{n-l-1}(Y(\beta_1,\beta_2) + Y(\beta_2,\beta_1)) = S_\perp^\sigma(Y_n^l + Y_n^{n-l}),$$

where β_1 (β_2) is the energy of the initial (final) quark divided by the energy of the hadron in a system with an infinite momentum, and $\beta_{12} = \beta_1 - \beta_2$ is the relative energy of the gluon in the initial state. The quantities β_1, β_2, and β_{12} can have either sign; a negative sign for β_i means that the corresponding particle is in the final state, rather than the initial state (or vice versa), and for $i = 1,2$ the quark is replaced by an antiquark.

It can be seen from (3) that only the even moments ($n = 0,2,4,...$) are important for deep inelastic ep scattering. Only certain combinations of the operators $R_{4l}^{\sigma\cdots}$, $R_{5l}^{\sigma\cdots}$, having the same charge parity, can mix with the operators $R_2^{\sigma\cdots}$ and $R_3^{\sigma_n\cdots}$. The matrix elements of these combinations are in fact given by (8) as integrals of the symmetric and antisymmetric parts of the function $Y(\beta_1, \beta_2)$.

4. The general method for deriving evolution equations for the parton distributions $O(\beta)$, $A(\beta)$, $C(\beta)$, and $Y(\beta_1, \beta_2)$ is set forth in Refs. 2. Since the results of our calculations are quite lengthy, we will give here only the equations for the moments determined by (8). From the Heisenberg equation of motion for the fields ψ, $(i\hat{\partial} - e\hat{A} - m)\psi(x) = 0$, we find

$$A_n = \frac{1}{n+1}\left[O_n + n C_n + \sum_{l=1}^{n-1} (n-l) Y_n^l \right], \tag{9}$$

where the quantities O_n, C_n, and Y_n^l satisfy the system of equations

$$\dot{O}_n = C_F\,(3 - 2S_n - 2S_{n+2})\,O_n, \quad \dot{C}_n = -4C_F S_n C_n,$$

$$\dot{Y}_n^l = \frac{4C_F}{l(l+1)(l+2)} C_n + Y_n^l \Big\{ 3C_F + 2\Big(C_F - \frac{C_v}{2}\Big)\Big(\frac{2(-1)^l}{l(l+1)(l+2)} - \frac{(-1)^{n-l}}{n-l+1} + \frac{1}{n}$$

$$- S_l - S_{n-l}\Big) + C_v \Big(\frac{2}{l(l+2)} - \frac{n+2}{(l+1)(n-l+1)} - 2S_l - 2S_{n-l} \Big)\Big]$$

$$- \sum_{k=1}^{l-1} Y_n^k \Big[2\Big(C_F - \frac{C_v}{2}\Big)(-1)^k \Big(\frac{2C_l^k}{l(l+1)(l+2)} + (-1)^l \frac{C_{n-1}^{k-1}}{C_{n-1}^{l-1}} \frac{n+l-k}{n(l-k)} \Big)$$

$$+ \frac{C_v (k+2)(k+1)}{(l+2)(l+1)(l-k)} \Big] + \sum_{k=l+1}^{n-1} Y_n^k \Big[2\Big(C_F - \frac{C_v}{2}\Big)(-1)^k \Big(- \frac{(-1)^n C_{n-l-1}^{k-l}}{n-l+1}$$

$$+ (-1)^l \frac{C_{n-1}^k}{C_{n-1}^l} \frac{n+k-l}{n(k-l)} \Big) + C_v \frac{(n-k)(n-k+1)}{(n-l)(n-l+1)(k-l)} \Big]. \tag{10}$$

By definition, $\dot{B} = dB/d\xi$, $\xi = b^{-1} \ln\Big(1 + b\frac{g^2}{16\pi^2} \ln\frac{q^2}{q_0^2}\Big)$, $b = \frac{11}{3} C_v - \frac{2}{3} n_b$, $C_F = \frac{N^2 - 1}{2N}$, $C_v = N$ [for the SU(N) group], n_b is the number of quark species,

$S_n = \sum_{k=1}^{n} (1/k)$, C_l^i are the binomial coefficients. The moments O_n correspond to the twist-2 matrix element of the operator $R_1^{...}$, and its anomalous dimensionality is the same, within a color factor, as in quantum electrodynamics.[6] The values of the moments A_0 and O_0 correspond to the Bjorken and Cottinham sum rules.[1] The expression for A_2 agrees with the result derived by Vaĭnsteĭn and Shuryak.[5] For the moment $n = 4$ the anomalous dimensionalities λ of the mixed operators $R_r (r \neq 1)$ are given approximately by $\lambda_1 = -11.1$, $\lambda_2 = -8.6$, $\lambda_3 = -13.55$, and $\lambda_4 = -27.8$ for the case of quantum chromodynamics ($N = 3$). Multiplicatively renormalized operators can easily be constructed from the solution of Eqs. (10). In a separate paper we intend to analyze the anomalous dimensionalities of operators which are singlet operators in the flavor quantum numbers. The approach outlined above could easily be generalized to other hard processes involving polarized particles.

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