On the mechanism of superconductivity in quasi-one-dimensional compounds

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Experimental data on the behavior of the critical temperature as a function of the concentration of lattice defects are used in formulating the concept of triplet pairing.

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Superconductivity has recently been discovered in a number of quasi-one-dimensional compounds under pressure ($TMTSF_2$–PF_6 and $TMTSF_2$–ClO_4, for example[1]). In several of these compounds studies have been made of the behavior under applied pressure of defects produced by intense x-ray bombardment. It was found that the defects strongly suppress the superconductivity, and when they reach atomic concentrations of the order of 10^{-4} the superconductivity is completely destroyed. Jerome and Schulz[1] have expressed the idea that the bombardment gives rise to unpaired electron spins, which play the role of magnetic impurities. True, those authors did note that even if all the defects produced are of this sort, they still would have too strong an effect (in ordinary three-dimensional superconductors the critical concentration of magnetic impurities is of the order of 1%). Of course, it could be assumed that the electron—impurity exchange interaction constant in these materials is 10 times greater, but one can see no reason why this should be so.

A possible way out of this disagreement lies in the assumption that triplet Cooper pairs, rather than singlet pairs, are formed in these materials. The gist of the argument is that ordinary nonmagnetic defects have as decisive an effect on triplet superconductivity as magnetic impurities have on singlet superconductivity, as was first shown by Larkin[2] for the three-dimensional case.

In order to give a quantitative description, let us make some assumptions. First, let us assume that the pairing takes place primarily on the same filament. As we know, a purely one-dimensional superconductivity is destroyed by fluctuations of the phase of the order parameter. We shall assume that the probability of hops from one filament to another is large enough to effectively suppress fluctuations but at the same time small enough to preserve the one-dimensional behavior of thermodynamic quantities which are not directly related to the hops (for the case of the behavior in a magnetic field this assumption is obviously incorrect, since the hops are important there). The feasibility of this approach has been demonstrated by Efetov and Larkin.[3]

The second assumption is that one may introduce a self-consistent field, as is done in BCS theory. It is known[4] that in the one-dimensional case there are not one, but two singular logarithms. As a result, it is not the "ladder" but rather a "parquet" situation that arises, and in this case the use of a self-consistent field is not justified. The ladder situation can be restored by the introduction of impurities, which do away with the

electron—hole logarithm associated with the formation of charge-density waves.[5,6] Of course, the impurities also affect the electron—electron logarithm (i.e., the superconductivity), the effect which we shall consider here. However, the first of the logarithms is eliminated by forward scattering, while the second one is affected only by backscattering. In view of this circumstance, we can consider only the electron—electron logarithm (although the effective interaction constant might be renormalized somewhat), and this justifies the introduction of the self-consistent field.

Including the potential and exchange interactions (without allowance for relativistic effects), we have the following interaction Hamiltonian for the electrons:

$$H_{ee} = \frac{1}{2} \int \psi_a^+(x)\, \psi_{a_1}(x)\, U_{aa_1\beta\beta_1}(x-x')\, \psi_\beta^+(x')\, \psi_{\beta_1}(x')\, dx\, dx', \tag{1}$$

where

$$U_{aa_1\beta\beta_1}(x-x') = U_1(x-x')\,\delta_{aa_1}\,\delta_{\beta\beta_1} + U_2(x-x')\,\mathbf{s}_{aa_1}\cdot\mathbf{s}_{\beta\beta_1}. \tag{2}$$

Here α and β are spin indices $(+,-)$, and $\mathbf{s}$ denotes a Pauli matrix. Since only the electrons in the vicinity of the momenta p_0 and $-p_0$ are important, we introduce, as in Ref. 7, the pseudospin indices 1 and 2 and the corresponding Pauli matrices $\boldsymbol{\sigma}$. Assuming that the interaction is weak, we keep only that part of the Hamiltonian which is important for the pairing, i.e., the part containing the combinations $\psi_{1+}+\psi_{2+}$, $\psi_{1-}\psi_{2-}$, $\psi_{1-}\psi_{2+}$, $\psi_{1+}\psi_{2-}$ and the corresponding combinations for ψ^+.

We now introduce the self-consistent pairing

$$\Delta_0 = 2^{-1/2}\left[\langle \psi_{1+}\psi_{2-}\rangle - \langle \psi_{1-}\psi_{2+}\rangle\right], \tag{3}$$

$$\Delta_{11} = \langle \psi_{1+}\psi_{2+}\rangle, \tag{4a}$$

$$\Delta_{1-1} = \langle \psi_{1-}\psi_{2-}\rangle, \tag{4b}$$

$$\Delta_{10} = 2^{-1/2}\left[\langle \psi_{1+}\psi_{2-}\rangle + \langle \psi_{1-}\psi_{2+}\rangle\right]. \tag{4c}$$

Here (3) gives the singlet and (4) the triplet pairing. The averages are taken in both the thermodynamic sense and over the random potential, consistent with long-range order.

The effective Hamiltonian becomes

$$H_{ee} = 2^{-1/2} g_0 \Delta_0 (\psi_{2-}^+\psi_{1+}^+ - \psi_{2+}^+\psi_{1-}^+) + g_1[\Delta_{11}\psi_{2+}^+\psi_{1+}^+ + \Delta_{1-1}\psi_{2-}^+\psi_{1-}^+ + 2^{-1/2}\Delta_{10}(\psi_{2-}^+\psi_{1+}^+ + \psi_{2+}^+\psi_{1-}^+)] + \text{c.c.}, \tag{5}$$

where

$$g_0 = U_1(0) + U_1(2p_0) - 3U_2(0) - 3U_2(2p_0), \tag{6}$$

$$g_1 = U_1(0) - U_1(2p_0) + U_2(0) - U_2(2p_0), \tag{7}$$

and $U_i(k)$ are the Fourier components of the functions in (2). Further, we include the

interaction with nonmagnetic impurities in the form of Gaussian random potentials η and ζ, as was done in Ref. 7.

We shall henceforth be interested only in the triplet pairing. To determine the critical temperature we find any of the Δ_1's in first order in H_{ee} and obtain the equation

$$g_1^{-1} = -T \sum_{\omega} \int_{-\infty}^{\infty} dx_1 < [G_{11}(xx_1\omega) G_{22}(xx_1-\omega) - G_{12}(xx_1\omega) G_{21}(xx_1-\omega)] >, \tag{8}$$

where the G's are the thermodynamic functions in the impurity potential. Expressing them in terms of G^R and G^A and substituting Eq. (4.16) from Ref. [illegible], we obtain an expression for these quantities in terms of the components of the S matrix, which must be averaged over the impurity potential. This last step is done as in Ref. 7.

The result depends only on the backscattering time τ_2. It cannot be written in analytical form for arbitrary ω. Because $|\omega|_{\min} = \pi T$, for $T\tau_2 \gg 1$ one also has $|\omega|\tau_2 \gg 1$. If, on the other hand, $T\tau_2 \ll 1$, then both $|\omega|\tau_2 \gg 1$ and $|\omega|\tau_2 \sim 1$ are important in the sum (8). The limiting expressions for the average in (8) for the cases $|\omega|\tau_2 \gg 1$ and $|\omega|\tau_2 \ll 1$ are of the form (the integrals over $x_1 > x$ and $x_1 < x$ are equal):

$$\int_{-\infty}^{x} <\ldots> dx_1 \approx \frac{\tau_2}{v} \frac{2}{\beta} \Big/ \left(1+\frac{4}{\beta}\right), \quad \beta = 4|\omega|\tau_2 \gg 1, \tag{9a}$$

$$\int_{-\infty}^{x} <\ldots> dx_1 \approx \frac{\tau_2}{v} 4\zeta(3)\beta, \quad \beta \ll 1. \tag{9b}$$

The first expression is correct to terms of order β^{-3}, inclusive.

In the limit of a pure metal, $T\tau_2 \to \infty$, we find from (8) for $g_1 < 0$ (Λ is the upper limit on ω, which is of order ε_F or ω_D):

$$T_{c1}^{(0)} = \frac{2\gamma\Lambda}{\pi} \exp\left(-\frac{2\pi v}{|g_1|}\right). \tag{10}$$

In the presence of impurities we find

$$\ln\frac{T_{c1}^{(0)}}{T_{c1}} = 4\pi\tau_2 T_{c1} \sum_{\omega>0} \left[\frac{2}{\beta} - v^2 \int_0^{\infty} <\ldots> dt\right], \tag{11}$$

where $t = (x - x_1)/v\tau_2$. In the sum in (11) terms up to $\beta \sim 4$ are important. Since the intersection of the asymptotic functions (9a) and (9b) occurs at $\beta = 0.101$, we can, without considerable error, use the asymptotic expression (9a) for arbitrary $T\tau_2$. The result is

$$\ln\frac{T_{c1}^{(0)}}{T_{c1}} = \psi\left(\frac{1}{2}+\rho\right) - \psi(1/2), \tag{12}$$

where $\psi(z) = (d/dz)\ln\Gamma(z)$ and $\rho = (2\pi\tau_2 T_{c1})^{-1}$.

The critical concentration, at which $T_{c1} = 0$, is determined by the relation

$$\tau_{2c}^{-1} = \pi T_{c1}^{(0)} / 2\gamma . \tag{13}$$

Near the critical concentration T_{c1} satisfies the relation

$$T_{c1} = (\sqrt{6}/\pi)\tau_{2c}^{-3/2}(\tau_2 - \tau_{2c})^{1/2},$$

in qualitative agreement with the experimental results of Ref. 1 (where, it should be noted, the dependence of T_{c1} on the longitudinal conductivity is given; however, this dependence is also determined solely by the backscattering, i.e., is proportional to τ_2). Further evidence in favor of the triplet-superconductivity model is the fact that at low pressures, where superconductivity does not occur, spin waves, rather than charge-density waves, arise. This might be due to the large value of the function $U_2(x)$ [Eq. (2)], which is important in both these cases.

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