On the frequency and temperature dependence of the conductivity near a metal-insulator transition

A. M. Finkel'shteĭn
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

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The metal-insulator transition is examined for interacting electrons in a disordered system. The renormalization-group equations are derived for two cases: transitions in a magnetic field and transitions in the presence of magnetic impurities. It is shown that in the critical region the frequency or temperature dependence of the conductivity is governed by a new invariant charge z.

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1. Experiments on disordered materials reveal that the dc conductivity goes to zero in a continuous manner[1,2] at the metal-insulator transition, and not in a jump. For this reason the theoretical treatment of this transition is currently proceeding in the spirit of the theory of second-order phase transitions. The construction of a theory incorporating both localization effects[3–6] and Coulomb correlations[7,8] has not yet been fully realized. An important attempt to construct a plausible scheme was undertaken by McMillan.[9] However, McMillan did not derive the renormalization-group equations which he used to describe the transition. In a recent paper[10] the present author proved the renormalizability and derived the renormalization-group equations for the case in which the interference corrections in the Copper channel are suppressed. Here it becomes clear that the McMillan scheme has certain shortcomings: The single-particle density of states should not appear among the renormalization-group charges. Furthermore, it was shown in Ref. 10 that the renormalization-group equations contain, in addition to the resistivity and Coulomb amplitudes, still another charge z, which arises on account of the renormalization of the frequency coefficient in the propagator for the diffusion of an electron of fixed energy:

$$< \psi_\epsilon^+ (r) \psi_{\epsilon+\omega}(r) \; \psi_{\epsilon+\omega}^+ (r') \psi_\epsilon (r') >_q = (Dq^2 - i\, z\, \omega)^{-1}. \tag{1}$$

It is shown in the present paper that z plays an important role in the description of the transition. In the critical region, the dependence of the conductivity and other properties on the frequency ω of the external field (or on the temperature T) is governed by the size of the parameter z near the fixed point of the renormalization-group equations.

2. For a finite frequency ω the characteristic electron diffusion length corresponding to (1) is

$$L_\omega = (D / z\, \omega)^{1/2}. \tag{2}$$

In the critical region the correlation length ξ is much greater than L_ω, and under these conditions the conductivity is given by

$$\sigma \sim e^2 L_\omega^{2-d} \quad , \tag{3}$$

where d is the dimensionality, and we have set $\hbar = 1$ throughout. To find $\sigma(\omega)$ we use the Einstein relation:

$$\sigma / e^2 = (\partial n / \partial \mu) D_e . \tag{4}$$

Here D_e is the diffusion coefficient for the total density of interacting electrons, and $\partial n/\partial \mu$ is the compressibility, which, in contrast to the case for the single-particle density of states, does not contain diffusion corrections.[10] As was shown in Ref. 10,

$$D_e = (1 + F_0) D; \quad \sigma / e^2 = (1 + F_0) \frac{\partial n}{\partial \mu} D, \tag{5}$$

where D is the quantity contained in the propagator (1), and F_0 is the constant from the Landau theory of the Fermi liquid. Solving (3) with allowance for what was said above, we find

$$\sigma(\omega) \sim e^2 \left(\frac{\partial n}{\partial \mu} z \omega \right)^{(d-2)/d} \tag{6}$$

The following types of behavior of z are possible in the critical region: (a) $z \to$ const. In this case $\sigma(\omega) \sim \omega^{1/3}$ for $d = 3$, just as in the case of noninteracting electrons,[11] where there is no renormalization of z whatsoever ($z = 1$). (b) $z \to 0$. In this case the dependenc of σ on ω is governed by the exponent ζ which describes how z approaches zero. (c) $z \to \infty$.

3. For describing the transition in a magnetic field or in the presence of magnetic impurities, the equations of the renormalization group can be derived in much the same way as was done on Ref. 10, since in both cases the Cooper channel is suppressed.[12,13] To incorporate the Coulomb interaction, two quantities are used: $\nu\Gamma$ and $\nu\Gamma_2$, where Γ is the small-angle and Γ_2 the large-angle scattering amplitude, and ν is the constant

$$\nu = \frac{1}{2} (1 + F_0) \frac{\partial n}{\partial \mu}.$$

It is important here that these two amplitudes correspond to different structures of the spin indices:

$$\Gamma \psi_p^{+\alpha} \psi_{\tilde{p}+k}^{+\beta} \psi_{\tilde{p}}^{\beta} \psi_{p+k}^{\alpha} + \Gamma_2 \psi_p^{+\alpha} \psi_{\tilde{p}+k}^{+\beta} \psi_{p+k}^{\beta} \psi_{\tilde{p}}^{\alpha}$$
$$= \psi_p^{+\alpha} \psi_{\tilde{p}+k}^{+\beta} \psi_{\tilde{p}}^{\gamma} \psi_{p+k}^{\delta} (\Gamma \delta_{\alpha\delta} \delta_{\beta\gamma} - \Gamma_2 \delta_{\alpha\gamma} \delta_{\beta\delta}).$$

For describing the transition, equations should be obtained for the following dimensionless quantities: z, $\nu\Gamma$, $\nu\Gamma_2$, and $L^{d-2}\sigma/e^2$, where L is the length variable, which changes in the renormalization process. There is an important relation among these quantities[10]:

$$z = 2\nu\Gamma - \nu\Gamma_2 . \tag{7}$$

Satisfaction of (7) ensures that the renormalization-group equations comply with the condition of conservation of the number of particles.[10]

a) Magnetic field. In the vicinity of the transition the region of interest is ω, $T < g_L eH/mc$, where g_L is the Landé factor.[1)] Here the Zeeman splitting leads to the cutoff of the pole in the diffusion propagator with opposite electron spin projections.[14,15] Taking this fact into account, we can derive the following equations in first order in $\epsilon = d - 2$:

$$\sigma / e^2 = G \left(\frac{2\pi}{L}\right)^{d-2} \frac{4k_d}{\pi}; \quad \frac{dG}{dx} = \epsilon G - \frac{1}{d}\left(2 + \frac{z + \nu\Gamma_2}{\nu\Gamma_2} \ln \frac{z}{z + \nu\Gamma_2}\right), \tag{8}$$

$$\frac{dz}{dx} = -\frac{1}{4G}(z - \nu\Gamma_2); \quad \frac{d\nu\Gamma_2}{dx} = -\frac{dz}{dx}; \quad \frac{d\nu\Gamma}{dx} = 0. \tag{9}$$

Here $x = \ln(L/\ell_0)$, where ℓ_0 is the mean free path, and $k_d = 2^{-d+1}\pi^{-d/2}/\Gamma((1/2)d)$. These equations have an unstable fixed point corresponding to the metal-insulator transition. It follows from (9) that in the critical region $z \to (1 + \nu\Gamma_2^0)/2$, where Γ_2^0 is the amplitude Γ without the diffusion corrections (the Fermi-liquid constant).

b) Magnetic impurities. Let

$$\sum_p \psi_p^{+\alpha} \psi_{p+q}^{\beta} = \delta_{\alpha\beta} w^0(q) + \vec{\sigma}_{\alpha\beta}\vec{w}(q),$$

where σ^i are the Pauli matrices. Magnetic impurities lead to the cutoff of the pole in the field correlator w,[6] so that w^0 is important. One is readily convinced that the Coulomb amplitudes in this case lead only to the combination $(2\nu\Gamma - \nu\Gamma_2)$, which, by virtue of (7), is equal to z. For this reason, the diffusion corrections in the presence of magnetic impurities should be particularly simple.[1)] To first order in ϵ we have $(x = \ln L/\ell_0)$:

$$\sigma/e^2 = G\left(\frac{2\pi}{L}\right)^{d-2} \frac{4k_d}{\pi}; \quad \frac{dG}{dx} = \epsilon G - \frac{1}{d}; \quad \frac{dz}{dx} = -\frac{1}{4G} z. \tag{10}$$

Thus, in the critical region we have $z \sim (\ell_0/L_\omega)^\zeta$, and from (2)–(5) we find

$$\sigma \sim \omega^{(d-2)/(d-\zeta)}. \tag{11}$$

To first order in ϵ we obtain the critical exponent $\zeta = 1/4G^* = \epsilon/2$.

4. We have been discussing the dependence of σ on the frequency of the external field for $\omega \gg T$. In the cases under consideration here the renormalization of the quantity G in Eqs. (8) and (10) is cut off at $L = \min(L_T, \lambda_\omega)$, where, by analogy with (2), the temperature length is $L_T = (D/zt)^{1/2}$. Therefore, if $\omega < T$ we have in the region of the metal-insulator transition

$$\sigma \sim T^{(d-2)/(d-\zeta)}, \tag{12}$$

where $\zeta = 0$ in the magnetic-field case and $\zeta = \epsilon/2$ in the magnetic-impurity case.

Without going into details, we conclude that the basic distinction between the theory of the present paper and that of McMillan[9] lies in the fact that the relationship between the energy and length scales is governed by the charge z, rather than by the single-particle density of states, as was proposed in Ref. 9.

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[1] Finkel'shteĭn[10] considered the behavior of a two-dimensional system in another temperature region: $eDH/c > T > g_L eH/mc$. The properties of the system when $g_L \ll 1$ turned out to be particularly interesting. In this case the logarithmic growth of the resistivity with decreasing T is replaced by a decline.

[2)] I am indebted to B. L. Al'tshuler and A. G. Aronov for pointing this out to me.

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