Power-law corrections to the quantum chromodynamics sum rules for charmonium

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The calculated $O(G^4)$ power-law corrections to the quantum chromodynamics sum rules for charmonium in the $J^{PC} = 1^{--}$ channel are discussed. Under the standard assumptions regarding the structure of the quantum chromodynamics vacuum, the incorporation of these corrections radically changes the shape of the theoretical curve for the ratio r_n, which is used in analyzing the low-lying states of charmonium by the quantum-chromodynamics sum-rule method.

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The technique of quantum-chromodynamics sum rules[1] is winning increasing popularity among theoreticians applying quantum chromodynamics to real processes. Since nonzero vacuum expectation values of the type $\langle\bar{\psi}\psi\rangle$, $\langle g^2 G^a_{\mu\nu} G^a_{\mu\nu}\rangle$ are included in the theory, it can be used to study the properties of hadrons (masses and lepton widths) associated with the long-range dynamics. In turn, by comparing the experimental data available with the theoretical expressions with the corrections for the higher-order vacuum expectation values ($\langle GGG\rangle$), $\langle GGGG\rangle$, etc.), one can draw conclusions regarding the magnitude of these vacuum expectation values and thus the structure of the quantum chromodynamics vacuum. In this connection a calculation of the higher-order power-law corrections to the quantum chromodynamics sum rules for charmonium is an important problem, although one which requires extremely laborious calculations. Our algorithm for calculating the power-law corrections was described in Refs. 2 and 3 and has been implemented in computer calculations using the SCHOONSHIP analytic calculation program.[4]

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We restrict the present paper to the vector channel, i.e., the J/ψ channel. We have derived the following expression for the moments $M_n^V = (-d/dQ^2)^n \pi^V(Q^2)/n!$ introduced in Ref. 1 [$\pi^V(Q^2)$ is the polarization operator for the current $\bar{c}\gamma^\mu c$]:

$$M_n^V = M_n^{(0)} \Big\{ 1 + a_n a_s - b_n < g^2 G^a_{\mu\nu} G^a_{\mu\nu} > + c_n < g^3 f_{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu} >$$

$$- d_n < g^4 j^a_\mu j^a_\mu > + \frac{4}{81} \frac{(n+3)!\,(2n+3)!!}{(n-1)!\,(2n+9)!!} < g^4 \,\mathrm{Sp}(\hat{G}_{\mu\nu} \hat{G}_{\nu\alpha} \hat{G}_{\alpha\beta} \hat{G}_{\beta\mu}) > \frac{1}{(4m_c^2)^4}$$

$$\times \left[\frac{23}{140} n^6 + \frac{4283}{420} n^5 + \frac{64747}{420} n^4 + \frac{12595}{12} n^3 + \frac{77234}{21} n^2 + \frac{226383}{35} n + 4512 \right] \Big\}, \qquad (1)$$

where $G = G^a \lambda^a/2$; the λ^a are the Gell-Mann matrices; $j^a_\mu = G^a_{\mu\nu;\nu}/g$; the coefficients $M_n^{(0)}$, a_n, and b_n are given in Ref. 1; and the coefficients c_n and d_n are given in Ref. 3. We are denoting the covariant derivative $D^{ab}_\alpha O^b$ by $O^a_{;\alpha}$.

Of the seven $O(G^4)$ contributions, we have written out explicitly in (1) only the one which is associated with the largest contribution in the system of estimates for the vacuum expectation values used below. Up to $n \sim 70$ the value of the corresponding coefficient is determined by not only the leading contribution in the nonrelativistic limit ($n \to \infty$), $O(n^6)$, but also by lower-n terms. The same is true of the contributions to (1) which we have not written out.

In the approach of Ref. 1 it is customary to analyze the ratio $r_n = M_n/M_{n-1}$, which tends in the limit $n \to \infty$ toward the value $1/M^2_{J/\psi}$. If the theory can tell us the value of r_n in the region where the experimental curve (Fig. 1) has already reached its asymptotic behavior (in practice, at $n \sim 6$), then we can work from the sign of $M_{j/\psi}$ to determine the magnitudes of the corresponding vacuum expectation values; conversely, if we know the vacuum expectation values we can estimate $M_{J/\psi}$ and the masses of

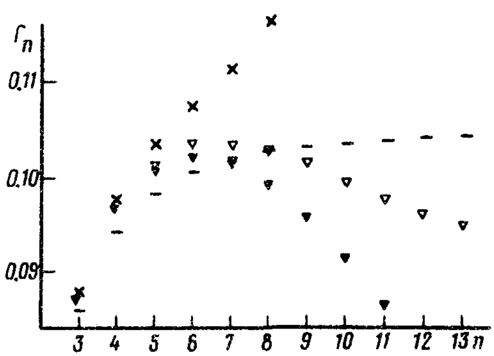

FIG. 1. - — Experimental; ▼—$O(G^2)$; ∇—$O(G^2) + O(G^3)$; ×—$O(G^2) + O(G^3) + O(G^4)$.

the low-lying states of charmonium in other channels. We therefore write out an explicit expression for r_n for $n = 6$.

$$r_6 = \frac{7}{36m_c^2}\{1 - 0.40\, a_s\,(2m_c) - 0.48\,\frac{\langle g^2 G^a_{\mu\nu} G^a_{\mu\nu}\rangle}{m_c^4}$$

$$+ \frac{1}{m_c^6}[0.59\langle g^3 f_{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu}\rangle - 2.08\langle g^4 j^a_\mu j^a_\mu\rangle]$$

$$+ \frac{1}{m_c^8}[3.63\langle g^4 \mathrm{Sp}(\hat G_{\mu\nu}\hat G_{\nu\alpha}\hat G_{\alpha\beta}\hat G_{\beta\mu})\rangle$$

$$-0.49\langle g^4\,\mathrm{Sp}(\hat G_{\mu\nu}\hat G_{\alpha\beta}\hat G_{\mu\nu}\hat G_{\alpha\beta})\rangle + 33.13\langle g^4 \mathrm{Sp}(\hat G_{\mu\nu}\hat G_{\nu\alpha}\hat G_{\alpha\beta}\hat G_{\beta\mu})\rangle -$$

$$-7.08\langle g^4 \mathrm{Sp}(G_{\mu\nu}G_{\alpha\beta}G_{\nu\alpha}G_{\beta\mu})\rangle - 7.07\langle g^5 f_{abc} G^a_{\mu\nu} j^b_\mu j^c_\nu\rangle - 2.92\langle g^3 f_{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu;\alpha\alpha}\rangle$$

$$+3.03\langle g^4 j^a_\mu j^a_{\mu;\alpha\alpha}\rangle - 0.36\langle g^2 G^a_{\mu\nu} G^a_{\mu\nu}\rangle^2]\}. \tag{2}$$

The $\langle g^2 G^2\rangle^2$ term arises because r_n is the ratio of quantities which are expanded in a series on $O(G^N)$.

It is clear from the very structure of expression (2) that this expansion does not reduce to a simple power series in, say, $\langle g^2 G^2\rangle/m_c^4$; the higher-order corrections depend on the particular model adopted for the quantum chromodynamics vacuum, i.e., on the method used to fix the values of the vacuum expectation values in (2). Shifman *et al.*[1] used the vacuum-dominance hypothesis and the approximation of a low-density instanton gas to fix the simplest vacuum expectation values. Accordingly, in plotting the curves in Fig. 1 we used some corresponding approximations: (a) For $\langle g^2 GG\rangle$ we used the value (0.83 GeV)^4 found in Ref. 1 by fitting the theoretical $O(G^2)$ curve to the experimental curve over an n interval as broad as possible (Fig. 1). (b) For $\langle jj\rangle$ and $\langle fGjj\rangle$ we used the vacuum-dominance hypothesis. For the ratio $\langle g\bar u(\sigma G)u\rangle/\langle \bar u u\rangle$ we used the value of 0.6 GeV^2, which agrees with choice d (specified below) and the previous estimate in Ref. 5. (c) We calculated $\langle g^3 fGGG\rangle$ in the approximation of a low-density instanton gas,[1] $\langle g^3 fG^3\rangle = (0.60\ \mathrm{GeV})^6$. (d) For the vacuum expectation values of operators containing $D^\alpha D_\alpha$ we used $\langle fGGG_{;\alpha\alpha}\rangle = M^2\langle fGGG\rangle$, $\langle jj_{;\alpha\alpha}\rangle = M^2\langle jj\rangle$, where the parameter $M^2 \equiv \langle GG_{;\alpha\alpha}\rangle/\langle GG\rangle$ is a measure of the mean virtuality of the vacuum gluons. From

$$\langle GG_{;\alpha\alpha}\rangle = 2\langle gfGGG\rangle - 2\langle g^2 jj\rangle$$

and choices (a)–(c) we find $M = 0.52$ GeV. Corresponding, from $\langle g\bar u(\sigma G)u\rangle = 2\langle \bar u u_{;\alpha\alpha}\rangle$ we find the estimate $\langle g\bar u(\sigma G)u\rangle = 2M^2\langle\bar u u\rangle$. (e) For the vacuum expectation values of the type $\langle g^4 \mathrm{Sp}(\hat G\hat G\hat G\hat G)\rangle$ we also use the vacuum-dominance hypothesis, which, when applied to the value $1152\langle g^4 \mathrm{Sp}\hat G^4\rangle/\langle g^2 G^2\rangle^2$, yields the values 110, 20, 47, and 29 for the first through fourth terms of the $O(m_c^{-8})$ contribution to (2), respectively. Some doubt has been raised in a series of papers[6–9] regarding the applicability of the vacuum-dominance hypothesis to calculations of vacuum expectation values of the type $\langle G^4 \rangle$. In particular, calculations from the instanton models[6–8] have yielded values five to ten times the results calculated by the vacuum-dominance hypothesis. (f) All the vacuum expectation values are normalized at $\mu^2 = -4m_c^2$, with $m_c = 1.26$ GeV, and the combinations $g^2G^2\rangle, \langle g^3fG^3\rangle, \langle g^4G^4\rangle$ and $\langle g\bar{\psi}\psi\rangle$ are treated as renormalization-invariant combinations (evidence in favor of the assumption that the corresponding anomalous dimensionalities are small comes from the fact that for instantons we have $G \sim 1/g$). Here $\Lambda = 0.1$ GeV; this value corresponds to $\alpha_s(2m_c) = 0.2$ and $\alpha_s(\mu_0) = 0.7$, where μ_0 is the normalization point, at which[1] $\langle \bar{u}u \rangle = -(0.24\ \text{GeV})^3$.

With the vacuum expectation values found by this method, the contributions to r_6 are distributed as follows:

$$r_6 = 0.1225\ \{1 - 0.080 - [0.086] + [0.007 + 0.003] + [0.012 - 3\times10^{-4} + 0.048 - 0.006 + 0.002 - 0.006 - 8\times10^{-4} - 0.013]. \quad (3)$$

It can be seen from (3) that the $O(G^4)$ correction is large at $n = 6$ and is due primarily to the vacuum expectation value $\langle g^4 \mathrm{Sp}\, (\hat{G}_{\mu\nu}\hat{G}_{\nu\alpha}\hat{G}_{\alpha\beta}\hat{G}_{\beta\mu})\rangle$, whose coefficient in (2) is much larger than the others (it should be noted that in this system of estimates all the vacuum expectation values of the operators of dimensionality 8 are similar in magnitude). The large value of this coefficient in turn results from the large coefficients of n^5, n^4, ... in (1).

If we increase all the vacuum expectation values of the type $\langle G^4 \rangle$ by an order of magnitude (i.e., if we adopt the model of Ref. 6), then the $O(G^4)$ correction at $n = 6$ is five times the $O(G^2)$ correction. Even at the moderate estimate based on the vacuum-dominance hypothesis the $O(G^4)$ corrections radically change the shape of the curve for r_n (Fig. 1). In particular, there is not even a hint of a plateau. This curve can be reconciled with the experimental curve only for $n = 2$–4. The value found for $\langle g^2G^2 \rangle$, as a result of the fit (at the fixed mass $m_c = 1.26$ GeV), is about twice the estimate given in Ref. 1.

In summary, if no mechanism is found to strongly suppress the vacuum expectation values of the type $\langle G^4 \rangle$ in comparison with the estimate from the vacuum-dominance hypothesis, we will be forced to question the reliability of the present applications of the method of Ref. 1 which are based on the $O(G^2)$ corrections exclusively. This attitude seems extremely realistic since, as mentioned earlier, in the existing models of the quantum-chromodynamics vacuum[6–8] the vacuum-dominance hypothesis leads to underestimates rather than overestimates of these vacuum expectation values.

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