Observation of an anomalous magnetoacoustic soliton in $KMnF_3$ crystals

Kh. G. Bogdanova, R. A. Bagautdinov, V. A. Golenishchev-Kutuzov, and V. P. Lukomskiĭ

Physicotechnical Institute, Academy of Sciences of the USSR, Kazan and Institute of Physics, Academy of Sciences of the USSR

(Submitted 4 April 1983)
Pis'ma Zh. Eksp. Teor. Fiz. **37**, No. 10, 483–486 (20 May 1983)

The results of observations of the anomalous propagation of an acoustic pulse in the frequency range between resonances of the nuclear magnetic subsystem in $KMnF_3$ are presented. It is shown that nonlinear propagation at a frequency corresponding to the point of inflection of the magnetoacoustic branch of the spectrum is described by a modified Korteweig–de Vries (KDV) equation.

PACS numbers: 72.55. + s, 43.35.Rw

The measurements were performed on a single-crystal specimen of $KMnF_3$ with length $l = 4.7$ mm at $T = 4.2$ K using the technique described in Ref. 1. Rectangular pulses with carrying frequency $f_H = 650$ MHz, $\tau_u = 0.3$ μs, and $P_a \leqslant 40$ Mv propagated along the [001] axis with a magnetic field H_0 in the (010) plane forming an angle ϕ with the [001] axis. The NMR frequencies of ^{55}Mn nuclei in a model with a two-sublattice antiferromagnet with skewed sublattices[2] in the spin-flop phase can be represented in the form

$$\omega_{1,2} = \omega_N \left(1 - \frac{2\,\omega_E\,\omega_{Ne}}{\Omega^2{}_{1,2}\,(H_0,\phi)} \right)^{1/2}, \qquad (1)$$

where $\omega_N = 2\pi$ 687 MHz is the undisplaced NMR frequency, ω_E, ω_{NE} are the frequencies of the exchange and hyperfine interactions, $\Omega_{1,2}(H_0,\phi)$ are the corresponding AFMR frequencies, expressions for which are presented in Ref. 2 for different orientations of H_0. Both frequencies in (1) depend on H_0 and ϕ, and $\omega_{1,2} \to \omega_N$ for $H_0 \to \infty$.

Anomalous propagation of the acoustic pulse was observed in a field H_0^x, located in the interval between the fields $H_{0_{1,2}}$ and the corresponding NMR resonances $(\omega_{1,2})$ at certain angles ϕ. For $P_a \leqslant 40$ mW and $H_0 < H_0^x$, a strong distortion of the trailing edge of the pulse passing through the specimen was observed (trace 2 in Fig. 1), while for $H_0 > H_0^x$, a distortion of the leading edge was observed (trace 3), which is evidently related to the cutoff of its spectrum at low or high frequencies. An intense, almost

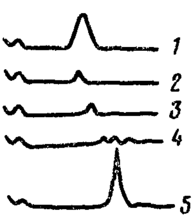

FIG. 1.

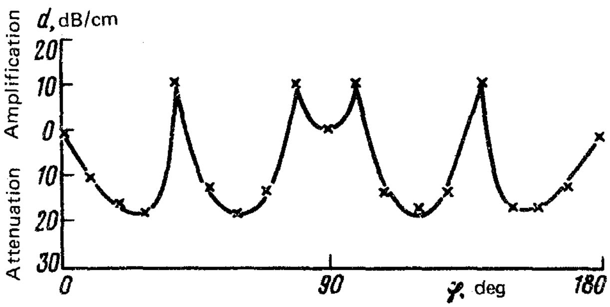

FIG. 2.

periodic modulation of the output signal, which was delayed compared to $t_3 = l/V_l$ [V_l is the velocity of the longitudinal sound wave (trace 4)], was observed at $H_0 = H_0^x$. For comparison, trace 1 shows the shape of the pulse with H_0 outside the interval $H_{0_{1,2}}$. With increasing power $P_a \gg 40$ mW, this periodically modulated pulse again becomes a single pulse with amplitude more than two times greater than the amplitude at $H_0 = 0$ and correspondingly compressed by a factor of 2 (trace 5). Figure 2 shows the angular dependence of the amplitude of the output pulse relative to $H_0 = 0$ (zero dB). For $\phi = 45°$ $H_0^x = 6900$ G, H_{0_1} and H_{0_2} correspond to 5525 and 7500 G, respectively. The periodicity is related to the cubic symmetry of the crystal and a period includes two critical angles $\phi^x = 40$ and 80°. Separation of the pulse into separate solitons was not observed up to $P_a \cong 1$ W.

These results can be explained theoretically within the framework of the analysis of nonlinear dynamics of a coupled electron-nuclear and elastic subsystems of the crystal taking into account the two NMR frequencies.[3] In the geometry that we used for the experiment, the dispersion law of the interacting nuclei and elastic waves in the quasistatic approximation ($\omega \sim \omega_{1,2} \ll \Omega_{1,2}$) can be represented in the form

$$k = \frac{\omega}{v_e}\left(1 + \frac{\delta_1^2}{\omega^2 - \omega_2^2} + \frac{\delta_2^2}{\omega^2 - \omega_1^2}\right)^{1/2}, \qquad (2)$$

where

$$\delta_{1,2} = \frac{c_0}{v_e}\,\omega_N (s_{1,2}\cdot r_{1,2})^{1/2}\Gamma_{1,2}(\phi)$$

are parameters characterizing the splitting of the branches of the spectrum, **K** is the wave vector, and the remaining notations are the same as in Ref. 1. The qualitative shape of the curves (2) is illustrated in Fig. 3.

The nonlinear evolution of the envelope of the magnetoacoustic pulse in the approximation of weak nonlinearity and dispersion

$$\left|\frac{\mathbf{m}_\perp}{\mathbf{m}_0}\right| \ll 1;\ \ \omega_2 - \omega_1, \qquad \delta_{1,2} \gg \tau_u^{-1}$$

near $\omega \sim \omega^x$ is described by the equation[4,5]

$$i[U_x + k' U_t - \frac{k'''}{6} U_{ttt} + q_2(|U|^2 U)_t] - \frac{1}{2} k'' U_{tt} + q_1 |U|^2 U = 0 \qquad (3)$$

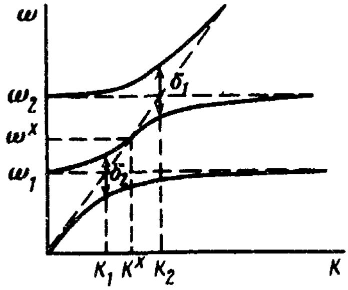

FIG. 3.

For $k''' = q_2 = 0$, Eq. (3) becomes the well-known nonlinear Schrödinger equation (NSE).[3,6] The expressions for the coefficients $q_{1,2}$ can be obtained with the help of the usual relaxation procedure, so that we do not present it here. At a frequency $\omega_H = w^x$, as can be seen from Fig. 3, $k''(\omega^x) = 0$, $k'''(\omega^x) < 0$. Within the framework of the linear theory, Eq. (3) is a linear KDV equation, whose solution is an Airy function transform of the input signal.[7] In our case, for a rectangular pulse with amplitude U_0 with $x = 1$ and $t \sim t_3$, we have

$$U(t, l) \approx U_0 A_i\left(\frac{t - t_3}{\tau_0}\right), \quad \tau_0 = \left(\frac{l}{2} k'''\right)^{1/3} \tag{4}$$

The modulation described by Eq. (4) agrees qualitatively with the experimentally observed modulation at this frequency (trace 4). For $k''' < 0$ the oscillating part of the envelope (4) turns out to be delocalized at $t > t_3$.

For $q_{1,2} \neq 0$, Hirota[8] obtained exact multisoliton solutions of Eq. (3) for the case $k'' \neq 0$. In the case of interest to us, $k'' = 0$, it can be shown that the time independent solution of (3) of the form $U(x) = U_0 \exp(+q_1 |U_0|^2 x)$ is stable relative to longitudinal perturbations. In the absence of modulation instability, the last term in (3) leads only to spatial modulation of the pulse. The larger nonlinear effect similar to the formation of a shock wave of the envelope is related to the term $\sim q_2$. Setting $q_1 = 0$ and using the fact that $k''0(\omega^x) = 0$, Eq. (3) transforms into the modified KDV equation. This equation is exactly integrable and its single-soliton solution describes the shape of the complex envelope of the pulse in the plane $x = 1$ with $\omega = \omega^x$

$$U(x, t) = U_0 e^{+iq_1 |U_0|^2 x} \operatorname{sech} \frac{t - \left(k' + \frac{U_0^2}{6} q_2\right) x}{\tau_s},$$

$$\tau_s = \left|\frac{k'''}{q_2}\right| \frac{1}{U_0}. \tag{5}$$

A characteristic property of the solution of (5) in comparison with the analogous solution of the NSE is the amplitude dependence of both the width and the velocity of the soliton. In addition, in spite of the similarity of the solitons described by these two equations, the physical mechanism for the formation of a KDV soliton is considerably different and is manifested in pure form only at the point $\omega = \omega^x$. On the other hand, this equation also has multisoliton solutions, which are not observed experimentally. This problem requires additional investigation.

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Translated by M. E. Alferieff
Edited by S. J. Amoretty