Joint analysis of data on pionic hydrogen and deuterium atoms

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Analysis of recent data on the shifts of the 1*s* levels of pionic hydrogen and deuterium atoms yields new values for the isosinglet and isotriplet πN scattering lengths. These new values contradict the results found from dispersion relations.

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Preliminary data obtained recently on the shifts of the 1*s* levels of pionic hydrogen and deterium atoms[1] can be used to find a new estimate of the isosinglet (b_0) and isotriplet (b_1) scattering lengths for *s*-wave pion-nucleon scattering. (We recall that the amplitude for pion-nucleon scattering at a zero energy is

$$a_{\pi N} = b_0 + b_1 \mathbf{t}\vec{\tau}, \tag{1}$$

where **t** and $\vec{\tau}$ are the operators representing the isospin of the pion and that of the nucleon.)

The energy shift of the 1*s* level of a pionic atom caused by the strong interaction is related in a simple way to the scattering length for the scattering of the pion by the corresponding nucleus[2] (which we denote by a):

$$\Delta E = -\frac{2\pi}{m} a \, |\psi_{1s}(0)|^2 \tag{2}$$

Here m is the reduced mass of the pion and the nucleus, and $\psi_{1s}(0)$ is the Coulomb wave function at the origin. Since a is small in comparison with the first Bohr radius of the system, $a/a_B \sim 10^{-3}$, the corrections to this relation for the $\pi^- p$ and $\pi^- d$ systems are negligibly small.[3] In turn, the scattering lengths for the scattering of pions by the lightest nuclei are related to b_0 and b_1. For scattering by the proton,

$$a_{\pi^- p} = b_0 - b_1. \tag{3}$$

The corresponding relationship for the deuteron is not as simple. A graphic method for calculating $a_{\pi d}$ was derived in Refs. 4–6. In a study of the multiple-scattering series it was shown that the single and twofold scattering of the pion by the nucleons are predominant; the departure of the amplitudes from the mass shell, the recoil of the nucleons (the deviation from the case of static nucleons), and the binding energy of the deuteron are all important. The *p*-wave part of the πN interaction and of higher-order rescatterings turns out to be extremely unimportant. The absorptive channel is also of minor importance in the real part of $a_{\pi d}$ and is taken from Ref. 7. What is important for our purpose at this point is that $a_{\pi d}$ can be expressed in terms of a linear and quadratic combination of b_0 and b_1, within small corrections. Taking this combination from Refs. 4–6 and using (2), we can then easily solve the inverse problem: that of finding b_0 and b_1 from the unknown $a_{\pi^- p}$ and $a_{\pi^- d_1}$.

Bovet *et al.*[1] found the shift of the 1*s* level of the pion-proton atom to be -12.1 ± 2.9 eV, while that for the pion-deuteron atom was found to be 4.8 ± 2.3 eV. These are the first measurements for the pionic atom of the proton, while the result for the case of pionic deuterium agrees well with the data of Ref. 8. The shifts given above correspond to the scattering lengths

$$a_{\pi^- p} = (0.151 \pm 0.035)\,\mu^{-1}; \qquad a_{\pi d} = (-0.052 \pm 0.025)\,\mu^{-1},$$

where μ is the mass of the π meson, $\mu^{-1} = 1.414$ F. Our analysis by the scheme outlined above yields

$$b_0 = (0.004 \pm 0.017)\,\mu^{-1}, \quad b_1 = (-0.147 \pm 0.035)\,\mu^{-1}. \tag{4}$$

Refined data on the shifts of the 1*s* levels of the pionic atoms of the proton and the deuteron could substantially reduce the error in the determination of b_0 and b_1.

Let us compare these results on b_0 and b_1 with results found previously. In several studies, b_0 and b_1 have been found by extrapolating data on πN scattering at energies from a few GeV to 30 MeV through the use of either dispersion relations or other analytic methods (see the compilation in Ref. 9, for example). For b_0 a range of values from $-0.030\mu^{-1}$ to $0.009\mu^{-1}$ has been found. The scatter in the values of b_1 is much smaller: from $-0.08\mu^{-1}$ to $-0.10\mu^{-1}$. The following are cited as the most plausible values in Ref. 10:

$$b_0 = (-0.013 \pm 0.003)\,\mu^{-1}; \qquad b_1 = (-0.092 \pm 0.001)\,\mu^{-1}.$$

We thus see an important contradiction with (4) in the value of b_1. The situation with regard to b_0 is less definite because of the large errors in its determination in (4).

In the analysis of data on the pionic atoms of heavier nuclei it is customary to use the Kisslinger-Ericson optical potential with one of the input parameters[10] $b_1 = -0.08\mu^{-1}$. The use of the value $b_1 = -0.12\mu^{-1}$, however, results in an equally good description of experimental data on the shifts and widths of *s* levels.[10] We thus find no direct contradiction of (4) here.

The data on the shift of the 1*s* level of the pionic atom of ^{4}He ($\Delta E \sim 76$–80 eV; Ref. 10) are not of critical importance for our purposes, since their description involves a quadratic combination of b_0 and b_1, which is similar to that in terms of which $a_{\pi d}$ is expressed, and if b_0 and b_1 are such that the pion-deuteron scattering length is reproduced well, then they will almost automatically give a good description of the π^4He scattering length also. For the so-called effective value b_0^{eff}, which was used in Ref. 11 to describe π^4He scattering and which is expressed in terms of b_0 and b_1, we find the estimate $(-0.029 \pm 0.011)\mu^{-1}$ from (4). To explain the experimental data we need $b_0^{\text{eff}} = (-0.023 \pm 0.001)\mu^{-1}$.

The pionic atom of ^{3}He is a more interesting case (the shift of the 1*s* level is from -30 to -35 eV, and the width is about 30 eV; Ref. 10). Comparison of the scattering lengths for the scattering of the pion by ^{3}He and ^{4}He yields[12]

$$\text{Re}\,\{\,a(\pi^-\,{}^4\text{He}) - a(\pi^-\,{}^3\text{He})\,\} = (-0.119 - -0.159)\,\mu^{-1}.$$

This value should be approximately equal to the $\pi^- n$ scattering length,[13] which is equal to $b_0 + b_1$, in excellent agreement with (4). On the other hand, a calculation of the π^3He scattering length in Ref. 12 with the help of the standard values of b_0 and b_1 (Ref. 9) yield a result which appears to be much too low if we take the absorptive channels into account, which are important in this case, since the level shift and the width are comparable in magnitude. The situation can be corrected only by increasing the modulus of b_1 substantially, in agreement with (4).

In summary, we have encountered a contradictory situation with regard to the *s*-wave πN scattering lengths. The new estimates of the scattering length in (4) agree with data on the pionic atoms of ^{3}He and ^{4}He but contradict the result found from an analysis of data on πN scattering through the use of dispersion relations.

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