MHD stability of a low-pressure plasma in an axisymmetric open system with an alternating-sign curvature

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A class of axisymmetric open configurations has been found. In them a long, thin plasma with $\beta \ll 1$ can be stable.

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1. Let us examine the MHD stability of a low-pressure plasma ($\beta = 8\pi p/B^2 \ll 1$, where B is the magnetic field) lying near a smooth but otherwise arbitrary surface of revolution (Fig. 1). There may be several magnetic mirrors along the field, so the pressure varies along the length of the system. In Fig. 1 a plasma with $p \neq 0$ fills regions 1 and 2; in connecting region 3 the pressure is negligibly low (but the conductivity is high, as in regions 1 and 2). The mirror ratios are assumed to be only slightly greater than unity, so that $p_\perp \gg p_\parallel$. To streamline the equations we assume that the electron pressure is much lower than the ion pressure.

2. In the limit $\beta \to 0$ the most dangerous flute waves are electrostatic waves: $\mathbf{E} = -\nabla\Phi$, where Φ remains constant along a magnetic line of force. The stability condition is[1–3]

$$W = \int \Phi^2 d\psi \int \Bigg\{ -\frac{\partial(p_\perp + p_\parallel)}{\partial\psi}\,\frac{1}{B}\,\frac{\partial B}{\partial\psi} + (p_\perp + p_\parallel)\left(\frac{1}{B}\,\frac{\partial B}{\partial\psi}\right)^2$$

$$+\, m_i \int \frac{B}{v_\parallel}\,\frac{\partial F}{\partial\epsilon}\left[\mu^2\left(\frac{\partial B}{\partial\psi}\right)^2 - \left(\frac{\displaystyle\int \frac{v_\parallel^2 + \mu B}{v_\parallel}\,\frac{1}{B}\,\frac{\partial B}{\partial\psi}\,dl}{\displaystyle\int \frac{dl}{v_\parallel}}\right)^2\right] d\mu\, d\epsilon \Bigg\} \frac{dl}{B} > 0. \qquad (1)$$

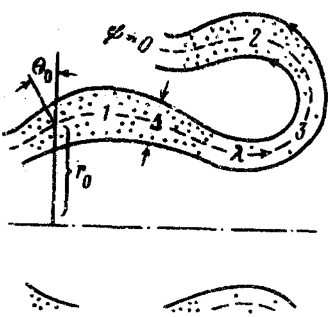

FIG. 1.

Here $F(\epsilon, \mu, \psi)$ is the unperturbed ion distribution function, which depends on $\epsilon = v^2/2$, $\mu = v_\perp^2/2B$, and the magnetic flux ψ; $v_\parallel = \sqrt{2(\epsilon - \mu B)}\cdot B$; and $p_\perp$ and $p_\parallel$ are functions of two coordinates, ψ and the "longitudinal coordinate" λ, whose surfaces of constant value run perpendicular to the $\psi = \text{const}$ surfaces. The integration over dl is carried out along a line of force; the element dl will be expressed in terms of ψ and λ below.

We restrict the discussion to the case in which the relative change in $\partial B/\partial\psi$ over the length of a single confinement system is $\lesssim p_\parallel/p_\perp$. In this case the term with $\partial F/\partial\epsilon$ simplifies: The terms with μ^2 "nearly" cancel out [the sum is $<p_\parallel[(1/B)(\partial B/\partial\psi)]^2$], and the leading term which remains, which is linear in μ, reduces to $2p_\perp[(1/B)(\partial B/\partial\psi)]^2$, so that

$$W = \int \Phi^2 d\psi \int \left[-\frac{\partial}{\partial\psi}\left(\frac{p_\perp + p_\parallel}{B^2}\right)\frac{\partial B}{\partial\psi} + \frac{p_\perp - p_\parallel}{B^3}\left(\frac{\partial B}{\partial\psi}\right)^2 \right] dl. \tag{2}$$

We thus find the sufficient condition for stability to be

$$\int \frac{\partial}{\partial\psi}\left(\frac{p_\perp + p_\parallel}{B^2}\right)\frac{\partial B}{\partial\psi}\, dl < 0. \tag{3}$$

3. Assuming that the field $\mathbf{B} = \mathbf{B}_0$ is given on the magnetic surface $\psi = 0$, along which we measure λ, we find $B(\psi,\lambda)$ near it. It is convenient to temporarily transform to some different orthogonal coordinates: w, which is measured in the meridional cross section along the normal to the curve $\psi = 0$; s, which is the coordinate of the base of the normal (Fig. 2); and the azimuthal angle φ. In terms of these coordinates the flux $\psi = rA_\varphi(\text{rot}A_\varphi\mathbf{e}_\varphi = \mathbf{B})$ can be written as the series[4]

$$\psi = r_0B_0w - \frac{r_0B_0}{2}\left(k_0 - \frac{1}{r_0}\cos\theta_0\right)w^2$$

$$+ \left\{\frac{r_0B_0}{3}\left(k_0^2 - \frac{k_0}{r_0}\cos\theta_0\right) - \frac{r_0}{6}\left[\frac{1}{r_0}(r_0B_0)'\right]'\right\}w^3 + \dots, \tag{4}$$

where $r_0(s)$ is the distance from the curve $\psi = 0$ to the axis, $k_0(s) = R_0^{-1}(s)$ is its curvature, $\theta_0(s)$ is the angle between the vector $\mathbf{w}$ and the radial direction (Fig. 1), and the prime denotes differentiation with respect to s. Evaluating the field components B_w and B_s, we find

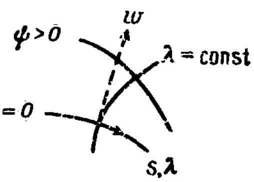

FIG. 2.

$$B = B_0 - k_0 B_0 w + \left\{ k_0^2 - \frac{1}{2B_0}\left[\frac{1}{r_0}(r_0B_0)'\right]' + \frac{1}{2}\frac{[(r_0B_0)']^2}{r_0^2B_0^2} \right\} B_0 w^2 + \dots . \tag{5}$$

For a transition to the coordinates ψ and λ, we use (4) to express w in terms of ψ: $w = \psi/r_0B_0 + \frac{1}{2}[k_0 - (1/r_0)\cos\theta_0](\psi/r_0B_0)^2 + \dots$. We also use the expansion $\lambda = s - \frac{1}{2}[(r_0B_0)'/r_0B_0]\,w^2 + \dots$ (which follows from the condition $\nabla\psi\nabla\lambda = 0$), and we then find $s(\lambda,\psi)$. As a result, we find

$$B = B_0 - k_0 B_0 \frac{\psi}{r_0B_0} + \frac{1}{2}\left\{ k_0^2 + \frac{k_0}{r_0}\cos\theta_0 + \frac{B_0'}{B_0}\frac{(r_0B_0)'}{r_0B_0} - \frac{1}{B_0}\left[\frac{1}{r_0}(r_0B_0)'\right]' + \frac{[(r_0B_0)']^2}{r_0^2B_0^2} \right\} B_0\left(\frac{\psi}{r_0B_0}\right)^2 + \dots , \tag{6}$$

where now B_0, r_0, k_0, and θ_0 are functions of λ.

4. The case $k_0 = 0$ corresponds to the Andreoletti-Furth confinement system,[5,6] in which there may be a shallow min$B$. The weak effect of a finite k_0 on the stability of such a confinement system was discussed in Ref. 6 (the effect is weak because we are dealing with the case in which the plasma occupies the region near the point with $k_0 = 0$).

5. Let us consider the case in which, in contrast with the Andreoletti-Furth case, the terms with $(r_0B_0)'$ in (6) are unimportant, but the terms with the curvature k_0 are important. We assume

$$r_0(\lambda)B_0(\lambda) = \text{const} \tag{7}$$

in the regions with $p \neq 0$. Substituting $\partial B/\partial\psi$ from (6) and $dl = (1 + k_0\psi/r_0B_0)d\lambda$ into (3), we find a sufficient condition for the stability of a thin plasma ($\delta w \equiv \Delta \ll r_0, R_0$):

$$\int \left(\frac{k_0}{r_0} - \frac{k_0\cos\theta_0}{r_0}\frac{\psi}{B_0r_0^2}\right)\frac{\partial}{\partial\psi}\frac{p_\perp + p_\parallel}{B^2}\,d\lambda > 0. \tag{8}$$

We require that the leading term in ψ in (8) vanish with an accuracy to Δ^2/r_0^2:

$$\int \frac{k_0}{r_0}\frac{p_\perp + p_\parallel}{B^2}\,d\lambda = 0. \tag{9}$$

This is possible if $k_0(\lambda)$ has an alternating sign. In this case, (8) reduces to

$$\int \frac{k_0\cos\theta_0}{r_0^2}\,\psi\,\frac{\partial}{\partial\psi}\frac{p_\perp + p_\parallel}{B^2}\,d\lambda < 0. \tag{10}$$

Under condition (9) and with (natural) pressure distributions such that $\psi(\partial/\partial\psi)(p/B^2) < 0$, the condition $k_0\cos\theta_0 > 0$ is sufficient for stability (this is the case shown in Fig. 1).

The reason for the stability of this configuration is that at $k_0 \cos\theta_0 > 0$ the decay of B as a function of ψ in the field-convexity direction occurs more slowly than linearly. The constant part of $\partial B/\partial\psi$ is of course the main part, so that in each of regions 1 and 2 one boundary is convex, and the isolated systems 1 and 2 are unstable. When the confinement systems are connected, on the other hand, $\partial B/\partial\psi$ is averaged out over the length, and under condition (9) the effects of the curvature cancel out. The effect which remains is equivalent to the presence of an average min$B$ (with a depth $\sim\Delta^2/r_0R_0$), since, as mentioned earlier, we have $\partial^2 B/\partial\psi^2 > 0$ in both confinement systems (although there is no well in either).

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