Inhomogeneous magnetoelectric effect

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A magnetically ordered medium should become electrically polarized near a magnetic inhomogeneity, in particular, in domain walls.

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If there is an equilibrium density of the electric polarization **P** which depends on the state of the magnetic subsystem in a homogeneous magnetic crystal, a "magnetoelectric effect" is said to occur. [1,2] This effect is intimately related to the magnetic symmetry of the system.[3] Neronova and Belov[4] have listed the magnetic symmetry classes which permit the magnetoelectric effect.

If, for some reason, a magnetic inhomogeneity arises in a magnetic crystal then the magnetic symmetry group of the crystal is lowered (to the unitary group for an inhomogeneity of a general type). As a result, an electric polarization **P**(**r**) arises in the vicinity of the magnetic inhomogeneity; the symmetry of the spatial distribution of this this polarization is determined by the symmetry of the magnetic inhomogeneity. We will call this effect the "inhomogeneous magnetoelectric effect," while the customary magnetoelectric effect[1–4] is the "homogeneous magnetoelectric effect." We wish to emphasize that this effect should occur in magnetic crystals of arbitrary symmetry (including crystals that do not permit the homogeneous magnetoelectric effect). The occurrence of an electric polarization because of an inhomogeneity (not necessarily magnetic in nature) is an extremely general effect. The flexoelectric effect in liquid crystals is a well known particular case.[5]

If the scale dimensions of the inhomogeneity, λ, are much larger than the dimensions of the elementary magnetic cell, the inhomogeneous magnetoelectric effect can be described phenomenologically. In the case of a ferromagnetic crystal, we need to introduce a coordinate-dependent magnetization density **M**(**r**) for this purpose. In general, magnetoelectric interactions have both short-range and long-range parts. In the present letter we consider only those cases in which the long-range component of the inhomogeneous magnetoelectric effect can be ignored. Restricting the discussion to terms of power no higher than the second in $M_\alpha (\alpha = x,y,z)$, and making use of the small parameter a/λ (a is the scale radius for the decay of the magnetoelectric interactions), we can then write the relationship between the polarization and the magnetization as follows:

$$P_i(\mathbf{r}) = f^{(0)}_{i,\alpha\beta} M_\alpha M_\beta + f^{(1)}_{ik,\alpha\beta} M_\alpha \frac{\partial M_\beta}{\partial x_k} + \frac{1}{2} \bar{f}^{(2)}_{ikj,\alpha\beta} \frac{\partial M_\alpha}{\partial x_k} \frac{\partial M_\beta}{\partial x_j} + \frac{1}{2} f^{(2)}_{ikj,\alpha\beta} M_\alpha \frac{\partial^2 M_\beta}{\partial x_k \partial x_j} \qquad (1)$$

The structure of the tensors $\hat{f}^{(0)}, \hat{f}^{(1)}, \hat{\hat{f}}^{(2)}, f^{(2)}$ is determined by the crystallographic class of the paramagnetic phase of the crystal. The tensor $\hat{f}^{(0)}$ differs from zero only in crystals that permit the homogeneous magnetoelectric effect.[2] If the nature of the magnetic inhomogeneity $\mathbf{M}(\mathbf{r})$ is known, we can determine the nature of the functions $P_\alpha(\mathbf{r})$ from (1). The components of the tensors $\hat{f}^{(0,1,2)}$ may be of relativistic, exchange-relativistic, and exchange origin. One of the most important manifestations of the inhomogeneous magnetoelectric effect is the appearance of an inhomogeneous electric polarization near boundaries between domain walls.

As an example we consider two types of plane 180° domain walls in ferromagnetic crystals of class $\mathbf{D}_{2h}$. We direct the z axis along the normal to the plane of the boundary, and the y axis runs along the easy-magnetization direction. Each of these directions coincides with a crystallographic axis.

(a) Bloch domain wall: $\mathbf{M} = M_0(\mathbf{e}_y \cos\theta - \mathbf{e}_x \sin\theta)$, $\theta = \theta(z)$. In this case, working in the leading approximation in a/λ, and using (2), we find the following expressions for the polarization components:

$$P_z(z) = f^{(1)}_{zz,yy} \frac{d}{dz} M_y^2 = -f^{(1)}_{zz,yy} M_0^2 \frac{d\theta}{dz} \sin 2\theta, \quad P_x = P_y = 0. \tag{2}$$

This polarization results from the space-charge density $\rho(z) = -dP_z/dz$. It is easy to see that $P_z(z)$ and $\rho(z)$ are nonzero only in a domain wall (i.e., at $d\theta/dz \neq 0$), and their average values over the thickness of the domain wall are precisely zero.

(b) Néel domain wall: $\mathbf{M} = M_0(\mathbf{e}_y \cos\theta - \mathbf{e}_z \sin\theta)$. In this case the coordinate dependence of P_z and P_x is the same as in (2). A difference is that in this case we have a polarization

$$P_y(z) = f^{(-)}\left(M_z \frac{dM_y}{dz} - M_y \frac{dM_z}{dz}\right) + f^{(+)}\left(M_z \frac{dM_y}{dz} + M_y \frac{dM_z}{dz}\right)$$

$$= M_0^2 \frac{d\theta}{dz}\left(f^{(-)} - f^{(+)} \cos 2\theta\right); \quad f^{(\pm)} = \frac{1}{2}\left(f^{(1)}_{zz,zy} \pm f^{(1)}_{zz,yz}\right). \tag{3}$$

Where the domain wall emerges at the surface of the sample (S) a charge appears, with a magnitude of $\pm \pi f^{(-)} M_0^2$ per unit length of the domain wall.

The application of an external magnetic field $\mathbf{H}$ makes an additional contribution to the inhomogeneous polarization density of orthorhombic ferromagnets; this contribution is described by the addition of terms of the type $f^{(H)}_{iz,\alpha\beta} H_\alpha dM_\beta/dz$ to (1).

If the macroscopic magnetic symmetry group of a crystal with domain walls consisting of operations that do not alter the magnetization distribution $\mathbf{M}(z)$ does not include operations that change the direction of the vector $\mathbf{e}_z$, an electric potential difference arises between domains:

$$\phi_1 - \phi_2 = 4\pi \int P_z(z)\, dz.$$

In particular, in crystals of orthorhombic symmetry we find

$$\phi_1 - \phi_2 = -8\pi M_0 f^{(H)}_{zz,\,yz} H_y \, .$$

In those crystals whose symmetry permits exchange terms $\mathbf{M}d^2\mathbf{M}/dz^2$ or eschange-relativistic terms[1)]

$$J^z_{yx} = M_y \frac{dM_x}{dz} - M_x \frac{dM_y}{dz}; \; J^z_{zy} = M_z \frac{dM_y}{dz} - M_y \frac{dM_z}{dz}$$

in expansion (1), the potential difference between domains is nonzero even if there is no external magnetic field. For example, exchange terms of this type are allowed in class $\mathbf{C}_{2v}$, while exchange-relativistic terms are allowed in class $\mathbf{C}_{2h}$ ($TiCr_2Se_4$, $TiCr_2Te_4$, Cr_3Te_4, etc.).

If the angle through which the vector $\mathbf{M}$ rotates in a domain wall is not equal to π, relativistic terms of the type dM_y^2/dz can make a significant contribution to the potential difference between domains.

If the magnetization vector $\mathbf{M}(\mathbf{r})$ is replaced by an antiferromagnetism vector $\mathbf{L}(\mathbf{r})$, the analysis above can be applied intact to antiferromagnets with two effective magnetic sublattices in the absence of an external magnetic field. In the present of a field $\mathbf{H}$, we need to consider in the expansion of $\mathbf{P}(\mathbf{r})$ terms of the type $H_\alpha^{dL}\beta/dz$, which change sign upon an interchange of sublattices.

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[1)]Bar'yakhtar and Yablonskiĭ[6] have shown that the presence of terms of the type $E_l J^k_{\beta\gamma}$ in the free energy of a magnetic material can induce helical magnetic structures in orthorhombic antiferromagnets when an electric field $\mathbf{E}$ is applied. We wish to thank A. A. Gorbatsevich for calling our attention to the possibility of the inverse effect due to these terms. An electric polarization of helical magnetic structures has been observed experimentally.[7]

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