Dynamics of smectic liquid crystals

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Nonlinear fluctuation effects in the dynamics of smectics are examined. It is shown that they lead to a low-frequency divergence $\propto \omega^{-1}$ in the kinetic coefficients. The logarithmic dependence of the coefficients in these divergences is calculated.

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In describing the nonlinear properties of smectic A, instead of the layer spacing u,[1] it is more convenient to use the function W, whose significance is that the equation $W = \text{const}$ determines the position of a layer of molecules. In terms of this function, the leading terms in the expansion of the free energy F have the form

$$F = \int d^3r \left(\frac{\beta}{8} ((\nabla W)^2 - l^{-2})^2 + \frac{\kappa}{2} (\nabla^2 W)^2 \right). \tag{1}$$

Here β and κ are the elastic moduli, and l is the distance between layers (period of modulation of the density). At equilibrium, $W_0 = z/l$, where the z axis is oriented perpendicular to the layers. As shown in Refs. 2 and 3, fluctuations around this equilibrium value lead to logarithmic renormalization of the moduli β and κ with the following dependence

$$\beta \sim L^{-4/5} \qquad \kappa \sim L^{2/5}. \tag{2}$$

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Here $L = \ln(\Lambda / \max(k_z, lk^2))$ is the large logarithm and Λ is the cutoff parameter.

It is natural to expect that strong long-wavelength fluctuations are also manifested in the low-frequency dynamics of smectic A. Mazenko *et al.*[4] calculated the first fluctuation corrections to the viscosity coefficients and found that they diverge at low frequencies as ω^{-1}. However, the higher-order corrections in perturbation theory also diverge in the same way. For this reason, it is clear that the analysis of the situation cannot be restricted to first-order perturbation theory. To include higher-order corrections in perturbation theory, we shall use the method developed in Ref. 5 by Khalatnikov, Sukhorukov, and one of the authors of this paper (V. L.)

Using the nonlinear equations of hydrodynamics of smectics, constructed in Ref. 6, we find the following nonlinear equation for W

$$\frac{\partial^2 W}{\partial t^2} = \frac{1}{\rho l^2} \frac{\delta F}{\delta W} + \frac{\partial}{\partial t} \frac{\Gamma}{T} \frac{\delta F}{\delta W} . \tag{3}$$

We note that Eq. (3) was obtained in the zeroth-order approximation with respect to the parameter $\beta / \rho l^4 c^2 \sim 10^{-3}$, where c is the velocity of sound. Only the first dissipative term was retained in Eq. (3) (with coefficient Γ), which, as will be evident below, is not fundamental.

The dynamic correlation functions containing W can be found by averaging with respect to the "microcanonical" distribution, corresponding to Eq. (3). The distribution function reduces in this case to a "functional" delta function, whose argument is Eq. (3), on whose right side we must add the random term $\partial / \partial t (\sqrt{\Gamma} \xi)$, where ξ is white noise. We raise the delta function into the exponent with the help of the auxiliary variable p and average over the white noise, integrating over ξ with the weight

$$\exp\left(-\frac{1}{4} \int dt\, d^3 r\, \xi^2 \right) .$$

As a result, we obtain the distribution function $\exp(iI)$, where I is the action with Lagrangian

$$\mathcal{L} = -p\, \partial^2 W / \partial t^2 - \frac{1}{2} (\nabla p \nabla W) a (l^2 (\nabla W)^2 - 1) - pb\nabla^4 W + ip\, \Sigma W + \frac{i}{2} p\, \Pi p . \tag{4}$$

Here the self-energy function Σ and the polarization operator Π are determined by the kinetic term

$$\Sigma = -i \frac{\rho l^2 \Gamma}{T} (a \nabla_z^2 - b\nabla^4) \frac{\partial}{\partial t} \qquad \Pi = -2\Gamma \partial^2 / \partial t^2 .$$

However, the first correction of perturbation theory, obtained by expanding with respect to the interaction in the Lagrangian (4), already strongly renormalizes Σ and Π, giving contributions of the form

$$\Sigma = -2ig \nabla^2 \partial / \partial t \qquad \Pi = -\tau \nabla^2 . \tag{5}$$

Thus, the fluctuation damping of the mode in smectics, which is attributed to the order parameter, is of the order of the frequency and, in addition, this is valid at all

frequencies, so that there is no region in which "normal" hydrodynamics is applicable. Renormalization of Lagrangian (4) with functions Σ, Π of the form (5) leads to the appearance of logarithmic corrections to the constants a, b, g, τ, so that the renormalization-group equations must be used in order to determine the long-wavelength behavior of these quantities.

We introduce the following single-particle Green's functions:

$$G(\omega, \mathbf{k}) = - < W(\omega, \mathbf{k}) p(-\omega, -\mathbf{k}) > \quad D(\omega, \mathbf{k}) = - < W(\omega, \mathbf{k}) W(-\omega, -\mathbf{k}) >. \quad (6)$$

The function G is the susceptibility of the system relative to the external force, while D is the pair correlation function. The poles of the G function, which lie in the lower half-plane, determine the spectrum of characteristic oscillations of the system. From (4), (5), and (6) we obtain

$$G = \frac{1}{i(\omega^2 - \eta^2) - 2g\omega k^2} \qquad D(\omega) = G(\omega)\, \Pi G(-\omega). \quad (7)$$

Here we use the notation $\eta^2 = ak_z^2 + bk^4$. At the pole of the G function the equation gives the following spectrum:

$$\omega_{\pm} = -igk^2 \pm \sqrt{\eta^2 - g^2 k^4}. \quad (8)$$

We note the following expression for the single-time correlation function, following from (8),

$$-\int \frac{d\omega}{2\pi} D = \tau / 4g\eta^2. \quad (9)$$

We now write out the renormalization-group equations for the quantities entering into Lagrangian (4). The interaction in it gives rise to the nonlinear term with coefficient a. It is convenient to represent the corresponding renormalization equations graphically (see Fig. 1). In these diagrams, the continuous line indicates the G function,

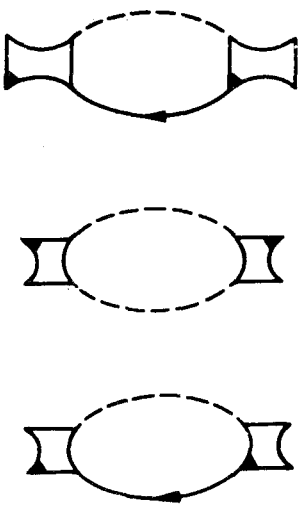

FIG. 1.

the dashed line indicates the D function, while the shape of the vertex represents the structure of the nonlinear term: the open ends correspond to ∇W and the shaded ends correspond to ∇p; the straight line segments connect quantities that form convolutions. The first diagram gives the renormalization of a, the second gives the renormalization of τ, and the third gives the renormalization of b and g. In addition, in the last two diagrams, the equilbrium value W_0 should be inserted at the external W ends. Integrating the expressions indicated over frequency and angles and going over to the differential equations, we find

$$a' = -\tau a^{3/2}/g b^{3/2} \qquad g' = \frac{1}{4}(1 + b/g^2)\tau a^{1/2}/b^{3/2}, \tag{10}$$

$$\tau' = \frac{1}{4}(1 + b/g^2)\tau^2 a^{1/2}/g b^{3/2} \qquad b' = \frac{1}{2}\tau a^{1/2}/g b^{1/2}.$$

Here the prime indicates differentiation with respect to the variable $Ll^2/128\pi$. It follows from (10) that

$$\tau/g = \text{const} \qquad dg^2/db = g^2/b + 1. \tag{11}$$

Taking this fact into account, we obtain from (10)

$$a \sim L^{-4/5} \qquad b \sim L^{2/5} \qquad g^2 = b \ln b/b_0. \tag{12}$$

The condition $\tau/g = \text{const}$ reflects the fluctuation-dissipation theorem and ensures, as is evident from (9) and (12), that the statistical limit is satisfied.

Corrections to the viscosity coefficients are determined by terms nonlinear with respect to W in the stress tensor, in which the "logarithmic" fluctuation corrections must be included. The fluctuation corrections to the viscosity coefficients are proportional to an integral of the form

$$\delta\eta \sim \int \frac{d\nu d^3q}{(2\pi)^4} q^4 \beta^2 D(\nu, \mathbf{q}) D(\omega + \nu, \mathbf{q}). \tag{13}$$

Equation (13) takes into account the fact that in the limit $a/c^2 \sim 10^{-3} \ll 1$, we can set $k = 0$ in the integrand. Carrying out the integral in (13), we find, after substituting (12), that

$$\delta\eta \sim \frac{1}{|\omega|} \frac{\tau^2\beta^2}{g^2 b^{3/2} a^{1/2}} \propto \frac{1}{|\omega|} L^{-9/5}. \tag{14}$$

This expression for the fluctuation part of the viscosity coefficients leads to an absorption coefficient for sound that is linear with respect to frequency. At low frequencies, this contribution to absorption is known to exceed the usual "linear" absorption $\propto \omega^2$. We note that this gives a qualitative explanation of the numerous experiments (see Refs. 7–9), in which a strong deviation of the absorption of sound in smectics from the law ω^2 was observed.

A detailed discussion of the problems examined above will be given in a separate paper.

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