Mass renormalization in Yang-Mills supersymmetry theories

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An $N = 2, 4$, Yang-Mills theory with a soft breaking of the expanded supersymmetry is analyzed. With $N = 4$, there are no divergences in the mass renormalization in any order. With $N = 2$, the mass and charge renormalization constants are the same and contain only single-loop divergences. Calculations are carried out to two loops.

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One of the attractive features of supersymmetry theories is the reduction in the number of parameters containing ultraviolet divergences. In the Wess-Zumino mass model,[1] for example, invariance with respect to supertransformations and a special form of the Lagrangian (related to the requirement of renormalizability) make the renormalization constants of the superfield, the mass, and the charge equal. There is one independent renormalization constant instead of the expected three.

Some recent three-loop calculations in Yang-Mills supersymmetry theories[2,3] indicate that ultraviolet divergences cancel out in this case. In the $N=4$ Yang-Mills supersymmetry theory there are no ultraviolet divergences, at least in the single-, two-, and three-loop approximations. In the $N=2$ theory there are divergences in the charge-renormalization constant at the single-loop level, but not at the two- and three-loop levels.[3] This tendency is expected to continue in the higher orders.[4]

Any supersymmetry theory claiming to be realistic must be a gauge theory and undoubtedly must contain mass fields. It is therefore interesting to examine the renormalization of not only the charge but also the masses.

Let us examine the renormalization of the masses of the matter fields in the theory describing the interaction of a vector $N=1$ superfield V with several chiral scalars of $N=1$ superfields S_i. Its effect is described by

$$S_r = S + S_m ,$$

where

$$S = \frac{1}{8g^2 c}\,\mathrm{Tr}\{\int dx d^2\theta\, W^{\alpha} W_{\alpha}\}$$

$$+\ \frac{2}{c}\,\mathrm{Tr}\{\int dx d^2\theta d^2\bar{\theta} \exp[-2gV]\bar{S}_i \exp[2gV] S_i\}$$

$$+\ \frac{4i\gamma}{3!\,c}\,\epsilon_{ijk}\,\mathrm{Tr}\{\int dx d^2\theta S_i [S_j, S_k] + \int dx d^2\bar{\theta}\bar{S}_i[\bar{S}_j, \bar{S}_k]\}$$

$$-\ \frac{1}{8\alpha c}\,\mathrm{Tr}\{\int dx d^2\bar{\theta} d^2\theta (D^2 V)(\bar{D}^2 V)\}$$

$$+\ \frac{2}{c}\,\mathrm{Tr}\{\int dx d^2\theta d^2\bar{\theta} (\bar{a}' - a') L_{gV}[(a + \bar{a}) + \mathrm{cth} L_{gV}(a - \bar{a})\},$$

$$S_m = -\sum_{i=1}^{n} \frac{m_i}{c}\,\mathrm{Tr}\{\int dx d^2\theta S_i S_i + \int dx d^2\bar{\theta}\bar{S}_i\bar{S}_i\}.$$

Here

$$L_x Y = [X, Y];\quad W^{\alpha} = (-D^2/4)(e^{-2gV} D^{\alpha} e^{2gV});$$

a is a ghost chiral superfield; α is the gauge parameter; and $i, j, k = 1, 2, \ldots, n$. All the fields transform in accordance with an associated representation of the gauge group:

$$V = V^a T^a, \qquad S_i = S_i^a T^a, \qquad a = a^a T^a$$

$$[T^a, T^b] = i f^{abc} T^c, \qquad f^{amn} f^{bmn} = c\delta^{ab}$$

If $m_i = 0$, for $\gamma = 0$ and $n = 1$ we have the $N=2$ Yang-Mills theory in terms of

$N=1$ superfields, while for $\gamma=g$ and $n=3$ we have the $N=4$ Yang-Mills theory in terms of $N=1$ superfields.[5] For $m_i \neq 0$, the $N=2$, 4 supersymmetry is broken in a soft fashion.

In the Wess-Zumino gauge,[6] we find the following results in terms of the ordinary fields from the equations of motion:

$$S_r = \int dx\, \mathcal{L},$$

$$\mathcal{L} = \mathrm{Tr}\Big\{ -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(D_\mu A_i)^2 + \frac{1}{2}(D_\mu B_i)^2 - \frac{1}{2}\bar{\varphi}_m i\hat{D}\varphi_m$$

$$+ \frac{ig}{2}\bar{\varphi}_m [\alpha^i_{ml}A_i + \gamma^5\beta^i_{ml}B_i, \varphi_l] + \frac{g^2}{4}([A_i, A_j]^2 + [B_i, B_j]^2$$

$$+ 2[A_i, B_j]^2) + \frac{i\gamma}{2}\sum_{i=1}^{n} m_i\epsilon_{ijk}(A_i[A_j, A_k] - A_i[B_j, B_k]$$

$$+ 2B_i[A_j, B_k] - \frac{1}{2}\sum_{i=1}^{n}(m_i^2A_i^2 + m_i^2B_i^2 \quad - m_i\bar{\psi}_i\psi_i)\Big\}\frac{1}{c}\,.$$

Here

$$\varphi = \begin{pmatrix}\psi_1\\ \psi_n\\ \lambda\end{pmatrix},$$

and the matrices α and β satisfy

$$[\alpha^i, \beta^j]_- = 0, \quad \{\alpha^i, \alpha^j\}_+ = \{\beta^i, \beta^j\}_+ = -2\delta^{ij},$$

$$\mathrm{tr}(\alpha^r\alpha^t) = \mathrm{tr}(\beta^r\beta^t) = -4\delta^{rt}.$$

Our purpose here is to examine the mass renormalization in the first few orders of perturbation theory,

$$m_{Ri} = Z_{S_i}m_i + \delta m_i$$

and to analyze the associated anomalous dimensionalities

$$\gamma_{m_i} = \frac{\partial \ln m_{Ri}}{\partial \ln\mu^2}\Big|_{m_i,\, g \text{ fixed}}.$$

According to a calculation of the divergence index,[7] there are no quadratic divergences in these theories; i.e.,

$$\delta m_i = 0\,.$$

Consequently, it is sufficient to know simply the renormalization constants for the wave functions of the fields S_i:

$$m_{Ri} = Z_{S_i}m_i = Z^{-1}_{m_i}m_i\,.$$

We have calculated Z_m both in the $N=1$ superfield formalism and in terms of ordinary fields, up to two loops inclusively. We used a regularization by the dimensional-reduction method and the 't Hooft minimum-subtraction method.[8] In the latter method, the introduction of masses does not alter the charge renormalization constant and thus does not alter the renormalization-group β function. Furthermore, there are no changes in the renormalization of the wave functions.

We can derive the relationship between β and γ in all orders of perturbation theory in the $N=4$ case. Since the renormalization constant for the $S_iS_jS_k$ vertex is unity according to a calculation of the divergence index,[7] we have

$$g_R^2 = Z_{S_i}^3 Z_{S_i^3}^{-2} g^2 = Z_{S_i}^3 g^2.$$

Differentiating the ratio

$$g^2/m_i^3 = g_R^2/m_{Ri}^3$$

with respect to $\ln \mu^2$ and holding g^2 and m_i fixed, we find

$$\beta(g_R^2) = 3g_R^2 \gamma_{m_i}(g_R^2).$$

Since we already know that $\beta(g_R^2)=0$ up to three loops inclusively, we also have

$$\gamma_{m_i}^{(3)}(g_R^2) = 0.$$

In the $N=2$ case, calculations in the superfield approach with two loops lead to

$$\gamma_{m_i}^{(2)}(g_R^2) = -2c\frac{g_R^2}{(4\pi)^2}.$$

Calculations in terms of the ordinary fields in the singleloop approximation yield the analytic functional dependence of γ_m on N. This number is related to the number of scalar fields, n:

$$N = n + 1,$$

which arises in a calculation of the Feynman diagrams containing a scalar trace. In this approximation we have

$$\gamma_{m_i}^{(1)}(g_R^2) = (N-4)c\frac{g_R^2}{(4\pi)^2}.$$

In the two-loop approximation, because of the interaction

$$\sim \sum_i m_i \epsilon_{ijk} \{A_i[A_j, A_k] - A_i[B_j, B_k] + 2B_i[A_j, B_k]\},$$

which occurs only for $N=4$, we cannot derive an analytic functional dependence on N in the obvious way. For $N=4$, the contribution of diagrams with this interaction to γ_{m_i} is $-12c^2g_R^4/(4\pi)^4$. Noting that in general these diagrams make a contribution $-6(N-2)c^2g_R^4/(4\pi)^4$, and adding the contributions of the other diagrams, in which the analytic functional dependence on N can be found in a trivial way, we find the following result for γ_{m_i} in the two-loop approximation:

$$\gamma_{m_i}^{(2)}(g_R^2) = (N-4)c\frac{g_R^2}{(4\pi)^2}\left[1-2c(N-2)\frac{g_R^2}{(4\pi)^2}\right].$$

Comparison with the result derived previously[3] for β,

$$\beta^{(2)}(g_R^2) = (N-4)c\,\frac{g_R^4}{(4\pi)^4}\left[1 - 2c(N-2)\,\frac{g_R^2}{(4\pi)^2}\right],$$

reveals

$$\beta^{(2)}(g_R^2) = \frac{g_R^2}{(4\pi)^2}\,\gamma_{m_i}^{(2)}(g_R^2).$$

We thus see that the charge and mass renormalization constants are related in not only the $N=4$ theory but also the $N=2$ theory. These constants are the same at least up to two loops inclusively. As shown above, this is true in all orders for $N=4$, and it probably remains true in the higher orders for $N=2$. These studies indicate that there may exist a finite four-dimensional quantum field theory containing massive fields.

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