Model-independent parameters of P-wave resonances in $p\alpha$ scattering

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Based on phase analysis of $p\alpha$ scattering, the vertex constants of $P_{3/2}$ and $P_{1/2}$ resonances, which determine the residues of renormalized Coulomb-nuclear partial scattering amplitudes of the poles on the second Riemann sheet corresponding to these resonances, are calculated.

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It is well known that the energy and width of $P_{3/2}$ and $P_{1/2}$ resonances in $N\alpha$ scattering, which are determined with the help of different resonance equations and, in particular, in R matrix analysis, contain large uncertainties and depend considerably on the model used.[1,2] In this connection, a model-independent determination, i.e., one based on the analytic properties of the S matrix, of the resonance parameters is of great importance: the position of the pole of the partial S-matrix on the second sheet of the Riemann surface in the complex k plane (k is the relative momentum of the particles) and the residue of this pole.[3] The significance of the latter quantity is analogous to the vertex constant for a bound state.[4,5] In the presence of a Coulomb interaction between particles, it must be kept in mind, however, that the analytic structure of the S-matrix changes radically.[6,7] We shall determine the vertex constant of the resonance using the same approach that we used previously to determine the vertex constant characterizing the virtual decay of the bound state into two fragments when there is a Coulomb interaction between fragments.[8] In this case, the parameters of the bound and resonant states are determined from a unified point of view.

Thus we shall examine the case of single-channel scattering of two identically charged spinless particles. The scattering amplitude can be represented in the form

$$A(k,\theta) = A_c(k,\theta) + \sum_{l=0}^{\infty} (2l+1)k^{2l}[h_l^{(c)}(k)]^{-2}\, T_l\, k^2)P_l(\cos\theta), \tag{1}$$

where $A_c(k,\theta)$ is the Rutherford amplitude θ is the scattering angle, $P_l(x)$ are Legendre polynomials, and $h_l^{(c)}(k) = l!\exp[(\lambda\pi/2\varkappa)\text{sign Re}k\,]/\Gamma(1+i\lambda/\varkappa)$ is the Coulomb-Yost function; $\lambda = \mu\alpha Z_1 Z_2$, μ is the reduced mass of the particles, Z_1 and Z_2 are the particle charges, and α is the fine structure constant. The renormalized Coulomb-nuclear amplitude $T_l(s)$ is an analog of the reduced partial scattering amplitude for scattering by a short range potential $A_l^{(n)}(s)$, since on the first (physical) sheet it has the same analytic structure as $A_l^{(n)}(s)$[6,7]. At the same time, the analytic properties of $T_l(s)$ and $A_l^{(n)}(s)$ on other sheets of the Riemann surface differ in the vicinity of the point $s = 0$. In the k plane, the amplitude $T_l(s)$ on the second sheet has a cut long the negative imaginary half-axis, an infinite sequence of zeroes at the points

$k = -i\lambda(n+1)(n=1,2,...)$, and poles at nearby points. If in the given partial wave there is a bound state (resonance), then the amplitude $T_l(s)$ has a pole on the first (second) sheet at $k = k_0$ and can thus determine the dimensionless constant R_l, related to the residue of this pole

$$R_l = i(-1)^{l+1} \lim_{k \to k_0} [(k-k_0) s^l T_l(s)]. \qquad (2)$$

For a bound state, the constant R_l is real and positive; it is related to the normalization factor in the asymptotic expression of the wave function a_l by the relation[8] $a_l^2 = 2\varkappa(l!)^{-2}\Gamma^2(l+1+\lambda/\varkappa)R_l$, where $\varkappa = -ik_0 \equiv \sqrt{2\mu\varepsilon}$, and ε is the binding energy, and when the Coulomb interaction is switched off, it is expressed in terms of the vertex constant G_l, determined in Ref. 4: $G_l^2 = 2\pi\varkappa\mu^{-2}R_1$. In the case of a resonance, the constant R_l is complex and, together with the quantity k_0, characterizes the resonant state in a model-independent manner. $T_l(s)$ satisfies the N/D representation, where as usual, the function $N_l(s)$ has only a left (dynamic) cut C_L, while $D_l(s)$ has only a right (unitary) cut C_R. If the contribution of inelastic channels is ignored, then the equation relating the functions $D_l(s)$ and $N_l(s)$ has the form

$$D_l(s) = 1 - \frac{s-s_0}{\pi} \int_{C_R} \frac{\rho_l(s') C_l^2(s') N_l(s')}{(s'-s)(s'-s_0)} ds', \qquad (3)$$

where $\rho_l(s) = S^{l+1/2}$ and $C_l^2(s) = |h_l^{(c)}(\sqrt{s})|^{-2}$ is the penetration factor. We determined the parameters of the P-wave resonances for states with total angular momentum $J = 1/2$ and $3/2$ in $p\alpha$ scattering $E_J^{(p\alpha)} = (K_{0J})^2/2\mu$ and $R_J^{(p\alpha)}$, using data from phase analysis.[9,10] The function $N(s)$ was parameterized in the form $N(s) = g^2(s)\xi(s)$,

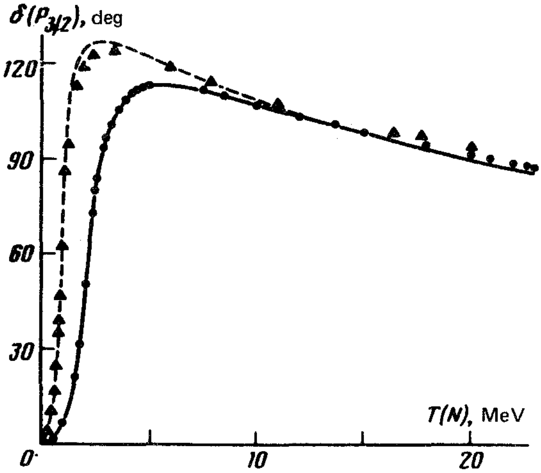

FIG. 1. Energy dependence of $P_{3/2}$ phase in $N\alpha$ scattering. The solid curve represents the calculation of the $P_{3/2}$ phase in $p\alpha$ scattering based on the parametrization indicated in the text (● indicates the data from phase analysis of $p\alpha$ scattering[9,10]). The dashed curve represents the calculation with the Coulomb interaction switched off (△ are the data from phase analysis of $n\alpha$ scattering[10,12]).

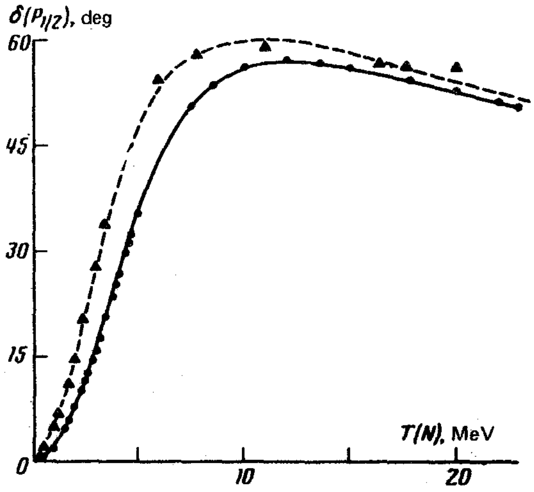

FIG. 2. Same as Fig. 1 for $P_{1/2}$ phase.

where $g(s) = \lambda (s+\beta^2)^{-2}$, while $\xi(s) = \exp[-(\lambda/\sqrt{s})\text{arc tg}(\sqrt{s}/\beta)]$ is a factor arising due to Coulomb renormalization of the dynamic cut for a separable interaction with form factor $g(k^2)$ (see, for example, Ref. 11). Equation (3) in this case determines the Coulomb-nuclear phase of the scattering and the analytic continuation of the amplitude $T_l(s)$ onto the second sheet of the Riemann surface. The parameters λ and β were chosen with the help of the χ^2 method from the measured phases of $p\alpha$ scattering.[9,10] As is evident from Figs. 1 and 2, the parametrization indicated (solid curves) reproduces well the energy dependence of $P_{3/2}$ and $P_{1/2}$ phases in the energy range examined. The values found by us for the vertex constants of P-wave resonances in $p\alpha$ scattering $R_J^{(p\alpha)} \equiv |R_J^{(p\alpha)}|\exp(\varphi_J^{(p\alpha)})$ constitute: $|R_{3/2}^{(p\alpha)}| = 0.30$; and $\varphi_{3/2}^{(p\alpha)} = -135.5°$ and $|R_{1/2}^{(p\alpha)}| = 0.31$; $\varphi_{1/2}^{(p\alpha)} = -177.9°$. In addition, $E_{3/2}^{(p\alpha)} = 1.65 - i\ 0.64$ MeV and $E_{1/2}^{(p\alpha)} = 2.70 - i\ 3.22$ MeV, which agrees within a few percent with the analysis in Ref. 1, wherein the positions of the resonance poles $E_J^{(p\alpha)}$ on the second sheet were determined by the method of polynomial expansion of the function of the effective radius. This method permits calculating the phases and parameters of resonances in $n\alpha$ scattering from $p\alpha$ scattering data. The results of the calculation of phases (dashed curves in Figs. 1 and 2) and resonance parameters with the Coulomb interaction switched off agree well with the measured phases[10,12] and parameters of P-wave resonances[1,2,13] in $n\alpha$ scattering.

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