Quantum-chromodynamics calculation of g_A

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The nucleon axial coupling constant g_A is calculated in nonperturbative quantum chromodynamics. The $g_A - 1$ renormalization stems from an interaction of the pion field with a quark condensate. The calculated results agree well with experiment.

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The method of quantum-chromodynamics sum rules proposed by Vaĭnshteĭn, Zakharov, and Shifman[1] is now used widely to describe the properties of hadrons. In particular, the masses of mesons[1] and baryons,[2,3] form factors, and meson coupling constants[4] have been calculated. Ioffe and Smilga[5] recently offered some sum rules for the polarization nucleon-current operator in an external electromagnetic field for use in calculating the magnetic moments of the nucleons. In the present letter we examine some corresponding sum rules in an axial external field, for use in calculating the axial coupling constant g_A. The possibility of using the external-field method for calculations was first raised by Ioffe and Smilga.[5]

1. We consider the polarization operator in an external field A_μ:

$$\Pi(q) = i \int d^4x\, e^{iqx} < T\{\eta(x), \bar{\eta}(0)\}>_A , \tag{1}$$

where

$$\eta(x) = \epsilon^{abc}[(u^a\ C d^b)u^c - (u^a C\gamma_5 d^b)\gamma_5 u^c] \tag{2}$$

is the quark current with the quantum numbers of the proton, C is the charge-conjugation matrix, and a, b, and c are the color indices. Polarization operator (1) in the absence of an external field was used in Ref. 2 to determine the mass of the proton. Analyzing the correlation functions in the axial external field A_μ is equivalent to analyzing the correlation functions in the ordinary vacuum of quantum chromodynamics, but with an additional term in the Lagrangian:

$$\Delta \mathcal{L} = \frac{1}{\sqrt{2}} (\bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d) A_\mu, \tag{3}$$

where A_μ is the axial external field, which is the third (neutral) component of the isotopic triplet. To evaluate g_A we must consider the approximation linear in A_μ.

Working from (3) and partial conservation of axial current, we find explicit expressions for the quark propagators and the vacuum expectation values, $\langle \psi^a_\alpha(x)\bar{\psi}^b_\beta(0)\rangle_A$ and $\langle \psi^a_\alpha(x)G^i_{\mu\nu}(y)\bar{\psi}^t_\beta(0)\rangle_A$, in the approximation linear in A_μ. The results are expressed in terms of the known vacuum expectation values and the pion constant $f_\pi = 133$ MeV.

By using an operator expansion we can calculate the polarization operator $\Pi(g)$ in an axial external field at $-q^2 \sim 1$ GeV^2. We can find g_A by taking into account the terms in the operator expansion up to $\langle \psi\psi \rangle^2$.

2. We now write the contribution of the physical states to polarization operator (1). The terms of interest here, which correspond to the diagram in Fig. 1a, are

$$<0|\eta|p><p|J^A_\mu|p><p|\bar{\eta}|0> A_\mu (q^2 - m^2)^{-2},$$

where

$$J^A_\mu = \frac{1}{\sqrt{2}} \bar{u}_p(q_1)[g_A(Q^2)\gamma_\mu \gamma_5 + f_1(Q^2) Q_\mu \gamma_5] u_p(q_2)|_{Q_\mu = q_{1\mu} - q_{2\mu} \to 0} \tag{4}$$

is the nucleon axial current. If we adopt the condition $Q_\mu A_\mu \equiv 0$, the term proportional to $Q_\mu \gamma_5$ vanishes, and the pole behavior of $f_1(Q^2)$ in the limit of $Q^2 \to 0$ presents no difficulties. In this case, in the limit of a static external field A_μ, in which we are interested, three tensor structures appear: $\hat{A}\gamma_5$, $(\hat{q}\hat{A} - \hat{A}\hat{q})\gamma_5$, $2(Aq)\hat{q}\gamma_5$. The last of these structures is the most perferable, for the following reasons: First, this structure contains the largest number of momenta, so that the contributions of the higher-lying baryon states and of the vacuum expectation values of the operators of higher dimensionalities to the sum rules are suppressed; second, it turns out that for this structure we can single out terms in the operator expansion corresponding to a sum rule with $g_A = 1$ and write sum rules for the quantity $g_A - 1$. In this paper we will consider only the structure $2(Aq)\hat{q}\gamma_5$.

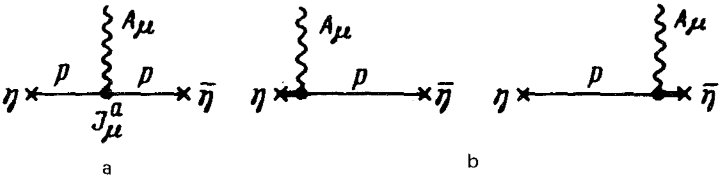

FIG. 1.

In addition to the double pole corresponding to the diagram in Fig. 1a, in which we are interested, there are also single-pole terms, corresponding to the diagrams in Fig. 1b. Ioffe and Smilga[5] have shown that the single-pole terms are not exponentially damped with respect to the two-pole terms in the Borel sum rules. In other words, in addition to the terms $(1/M^2)\exp(-m^2/M^2)$, which correspond to the double pole, there are also terms $\exp(-m^2/M^2)$, which correspond to single poles and which must be taken into account in order to find the correct value of g_A.

3. A calculation of the polarization operator leads to following Borel sum rules for the structure $2(Aq)\hat{q}\gamma_5$:

$$\frac{1}{8}M^4+\frac{g^2<G^2>}{32}+\frac{(2\pi)^4<\bar{\psi}\psi>^2}{M^2}\left(\frac{1}{6}+\frac{1}{9}\right)+\frac{(2\pi)^2 f_\pi^2\, m_1^2}{4\cdot 18}$$

$$=\left(\frac{\tilde{\beta}^2 g_A}{M^2}+C\right)\exp(-m^2/M^2)+\sum_i\left(A_i/M^2+D_i\right)\exp(-m_i^2/M^2), \qquad (5)$$

where

$$<0|\eta|p>=\tilde{\beta}/(2\pi)^2\gamma_5 v(q), \quad (\hat{q}-m)v(q)=0,$$

and C, A_i, and D_i are unknown constants. We might note that the value of $m_1^2=0.8$ GeV^2 found by Novikov[6] appears in the expression for the matrix element, $g\langle 0|\bar{u}\tilde{G}_{\mu\nu}\gamma_\nu d\,|\pi\rangle = i m_1^2/4 f_\pi q_\mu$. Comparing (5) with the sum rules found in Refs. 2 and 3, we see that some of the terms of the operator expansion correspond to a sum rule with $g_A=1$.

The sum rule in Ref. 2 in which we are interest here is

$$\frac{1}{8}M^4+\frac{g^2<G^2>}{32}+\frac{(2\pi)^4<\bar{\psi}\psi>^2}{6M^2}=\frac{\tilde{\beta}^2}{M^2}\exp(-m^2/M^2)$$

$$+\text{ (contribution of higher-lying states).} \qquad (6)$$

It can be shown that the terms in the operator expansion which are common to the two sum rules correspond to diagrams in which the external field (or pion) interacts with hard quarks. We can thus formulate some sum rules directly for the quantity g_A-1. Working from (5) and (6), making use of the anomalous dimensionalities of the baryon currents, and of the terms of the operator expansion, and ignoring higher-lying states, we find some rules for g_A-1:

$$(g_A-1)+C'M^2=\frac{1}{\tilde{\beta}^2}\exp(m^2/M^2)\left[\frac{(2\pi)^4<\bar{\psi}\psi>^2}{9}L^{4/9}+\frac{f_\pi^2\,(2\pi)^2 m_1^2}{4\cdot 18}L^{-8/9}\right], \qquad (7)$$

where[2,3]

$$L=\left(\frac{\alpha_s(\mu)}{\alpha_s(M)}\right),\quad \mu=0.5\text{ GeV}\quad\text{and}\quad \tilde{\beta}^2=0.35\text{ GeV}^6.$$

Analysis of sum rules (7) yields the value $g_A-1=0.3\pm0.05$, where the error is determined basically by the uncertainty in the value of the residue $\tilde{\beta}^2$. The quantity C' turns out to be about -0.03 ± 0.02 GeV^{-2}. The apparent reason for the numerically small value of this quantity is the fact that the widths for the decay of the higher-lying nucleon resonances into $p\pi$ are small.

The value found for $g_A - 1$ is in excellent agreement with experiment. It should be noted that by virtue of the Goldberger-Treiman relation a calculation of g_A is also a calculation of the pion-nucleon coupling constant $g_{\pi NN}$, and by using the Adler-Weisberger sum rule we can calculate the constant of the transition $\Delta \to p\pi$.

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