Equations of motion of variable-mass particles and quark confinement

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Relativistic and nonrelativistic equations of motion are written for particles whose mass depends on the coordinates. It is shown that quark confinement and the existence of quarkonium are possible in potential models only if the quark mass increases at large distances.

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Our primary purposes here are to write equations for "particles" whose mass depends on the coordinates and to briefly discuss some consequences and applications of these equations. These physical entities cannot, of course, be free particles. Quarks, which never appear in a free state, are possible candidates. In addition, we do not rule out the possibility that the equations derived here may also prove useful, in particular, for describing effects in solid state physics (in two-band semiconductors, for example).

We consider the motion of a relativistic particle in a centrally symmetric electrostatic field. If the mass of the particle varies along the coordinate, the relativistic Hamiltonian of the system in the potential $V_0(r)$ is

$$H = \vec{\alpha}\mathbf{p} + \beta m(r) + V_0(r), \tag{1}$$

where the dependence of the mass m on the distance r is indicated explicitly. For convenience in our subsequent transition to the nonrelativistic limit we single out this dependence:

$$m = m_0 + V_m(r), \tag{2}$$

where m_0 is the mass of the "free particle" [or the mass at the point $r = r_0$, where $V_m(r) = 0$].

The system of equations for the radial wave functions is

$$\begin{cases} F' + \kappa r^{-1}F - (\epsilon + m_0 + V_m(r) - V_0(r))G = 0, \\ G' - \kappa r^{-1}G + (\epsilon - m_0 - V_m(r) - V_0(r))F = 0, \end{cases} \tag{3}$$

where $F(G)$ is the upper (lower) spinor, ϵ is the total energy, and $\kappa = l$ at $j = l - 1/2$ or $\kappa = -(l+1)$ at $j = l + 1/2$ (j and l are respectively the total and orbital angular momenta).

Eliminating one of the functions, in the standard approach, we find a second-order equation,

$$F'' + [(\epsilon - V_0)^2 - (m_0 + V_m)^2 - l(l+1)r^{-2}] \\ F - (V_m' - V_0')(\epsilon + m_0 + V_m - V_0)^{-1}(F' + \kappa r^{-1}F) = 0, \tag{4}$$

and the function G is found by solving this equation and using the formula

$$G=(\epsilon+m_0+V_m-V_0)^{-1}(F'+\kappa r^{-1}F). \tag{5}$$

We understand the "nonrelativistic limit" of Eq. (4) to mean the case

$$m_0 \gg E+V_m-V_0, \tag{6}$$

where $E=\epsilon-m_0$ is the nonrelativistic energy.[1] In the lowest-order approximation we find the Schrödinger equation

$$F''-[l(l+1)r^{-2}-2m_0(E-V_m-V_0)]F=0, \tag{7}$$

where the role of the potential is played by the sum

$$V(r)=V_m(r)+V_0(r). \tag{8}$$

We note that V_m is a Lorentz scalar, while V_0 is a component of a Lorentz vector.

The potential used in the Schrödinger equation for variable-mass particles is thus determined by the sum of the potential energy in the external field and the increase in the mass of this particle.

These components of the potential arise separately only in the next highest approximation order, where the first few relativistic corrections are taken into account:

$$F''-[l(l+1)r^{-2}-2m_0(E-V_m-V_0)]F$$
$$-(V_m'-V_0')(2m_0)^{-1}(F'+\kappa r^{-1}F)+[(E-V_0)^2-V_m^2]F=0. \tag{9}$$

The third term gives the potential-energy correction and the spin-orbit interaction, modified by the mass gradient, while the last term is the sum of the classical relativistic mass correction and a new term which is proportional to $V_m^2(r)$.

All further approximations are easily derived from (4) by expanding the second factor in the last term. For specific calculations based on Eq. (9) we must also take into account the change in the normalization of F when we switch from (7) to (9).

If the nonrelativistic potential is determined by the sum V_m+V_0, then the difference $V_m'-V_0'$ is totally responsible for the fine structure of the levels, as can be seen from (9). Accordingly, information on both the posiition and fine structure of the levels can be found by singling out the separate roles of V_m and V_0.

The most realistic candidates for the role of the particles described by these equations are quarks. We note the three following properties of quarks:

1. At short range their mass (the current mass) is lower than at long range (the constituent mass).

2. They do not appear in a free state.

3. Bound systems of quarks exist (bound states of heavy quarks are described well by nonrelativistic potential models).[1-3]

The last two of these properties are usually taken into account in a phenomenological way in that the interaction potential of the quarks increases without bound at long range. In the many-particle problem, however, this growth should cause a creation of pairs and a general instability analogous to the instability of the relativistic problem of an overly steep (near the origin of coordinates) repulsive wall, which leads to a decrease toward the center (in contrast with the nonrelativistic case).

It is clear from an inspection of Eqs. (3) that they do not give rise to bound states if the asymptotic growth of the coefficients in the last terms is determined by the external potential V_0, since only unnormalizable oscillatory solutions exist in this case. Bound states can exist only for systems for which the asymptotic behavior of these coefficients is determined by the increase in the mass, i.e., $V_m(r)$. Only in this case do the equations allow an exponentially damped asymptotic behavior of the wave functions at infinity.

As an example[2)] we write this asymptotic behavior for linearly increasing potentials,

$$V_m \approx V_0 \approx kr \quad \text{as} \quad r \to \infty \tag{10}$$

with a cancellation of the type

$$V_m - V_0 = qr^\nu \, (0 < \nu < 1, \, q > 0). \tag{11}$$

The solutions of system (3) are

$$F|_{r\to\infty} \sim \exp\left\{ -\frac{4}{3+\nu} \sqrt{\frac{kq}{2}} \, r^{\frac{3+\nu}{2}} \right\} \tag{12}$$

(in the limiting cases $\nu = 1$ and $\nu = 0$ we find the solutions as in the case of a one-dimensional oscillator or as in the problem of a uniform field, respectively). The nonrelativistic equation does not yield exact asymptotic expressions, since the condition that $V_m - V_0$ be small in comparison with m_0 is violated (although confinement does occur, of course).

In summary, we may say that a description of systems in a centrally symmetric field with potentials which increase without bound at long range can be systematic only for "particles" whose mass also increases with increasing distance. If we apply these arguments to quarks we conclude that the quark confinement results from the increase in the mass of the quarks as they move away from each other. In general, an increase in the "external" potential $V_0(r)$ is not required. The increase in the quark mass is itself the source of the confining potential. The dynamic reason for the mass increase can be determined, of course, only by solving the many-particle problem. The relativistic description of these "particles" is quite different from the usual description, and the nonrelativistic limit can be taken only in a bounded volume.

[1)]In general, this inequality holds over a bounded interval of distances.
[2)]Some particular cases were also discussed in Refs. 4 and 5.

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