Pis'ma v ZhETF, vol. 77, iss. 2, pp. 68 – 72 

# Pion distribution amplitudes within the instanton model of QCD vacuum

*A. E. Dorokhov*

*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia*

Submitted 18 December 2002
Resubmitted 25 December

Pion transition form factor for the process $\gamma^\star\gamma^\star \to \pi^0$ at space-like values of photon momenta is calculated within the effective quark-meson model with the interaction induced by instanton exchange. The leading and next-to-leading order power asymptotics of the form factor and the relation between the light-cone pion distribution amplitudes of twists 2 and 4 and the dynamically generated quark mass are found.

PACS: 13.40.Gp

The pion form factor $M_{\pi^0}(q_1^2,q_2^2)$ for the transition process $\gamma^\star(q_1)\gamma^\star(q_2) \to \pi^0(p)$, where $q_1$ and $q_2$ are photon momenta, is related to fundamental properties of QCD dynamics at low and high energies. At zero photon virtualities the observed value of the width for the two-photon decay of the $\pi_0-$meson

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{e^2 m_{\pi_0}^3}{64\pi} M_{\pi^0}^2(0,0) = 7.79(56)\ \text{eV}, \quad (1)$$

is consistent with the theoretical prediction due to the chiral anomaly for $\pi_0$

$$M_{\pi^0}(0,0) = (4\pi^2 f_\pi)^{-1}, \quad (2)$$

where $f_\pi = 92.4$ MeV is the pion weak decay constant.

The existing experimental data from CELLO [1] and CLEO [2] Collaborations on the form factor $M_{\pi^0}$ for one photon being almost real, $q_2^2 \approx 0$, with the virtuality of the other photon scanned up to 8 GeV$^2$, can be fitted by a monopole form factor:

$$M_{\pi^0}(q_1^2=-Q^2, q_2^2=0)\big|_{fit} = \frac{g_{\pi\gamma\gamma}}{1+Q^2/\Lambda_\pi^2}, \qquad \Lambda_\pi \simeq 0.77\ \text{GeV}, \quad (3)$$

where $g_{\pi\gamma\gamma} = 0.275$ GeV$^{-1}$ is the two-photon pion decay constant. The large $Q^2$ behavior of the form factor (3) is in agreement with the lowest order perturbative QCD (pQCD) prediction [3]

$$M_{\pi^0}(q_1^2,q_2^2)\big|_{Q^2\to\infty} = J^{(2)}(\omega)\frac{1}{Q^2} + J^{(4)}(\omega)\frac{1}{Q^4} + O\left(\frac{\alpha_s}{\pi}\right) + O\left(\frac{1}{Q^6}\right), \quad (4)$$

where the leading (LO) and next-to-leading (NLO) order asymptotic coefficients $J(\omega)$ are expressed in terms of the light-cone pion distribution amplitudes (DA), $\varphi_\pi(x)$:

$$J^{(2)}(\omega) = \frac{4}{3} f_\pi \int_0^1 dx \frac{\varphi_\pi^{(2)}(x)}{1-\omega^2(2x-1)^2},$$
$$J^{(4)}(\omega) = \frac{4}{3} f_\pi \Delta^2 \int_0^1 dx \frac{1+\omega^2(2x-1)^2}{[1-\omega^2(2x-1)^2]^2}\varphi_\pi^{(4)}(x). \quad (5)$$

In the above expressions $Q^2 = -(q_1^2+q_2^2) \geq 0$ is the total virtuality of photons and $\omega = (q_1^2-q_2^2)/(q_1^2+q_2^2)$ is the asymmetry in their distribution. The distribution amplitudes are normalized as $\int_0^1 dx\varphi_\pi(x) = 1$ and the parameter $\Delta^2$ characterizes the scale of the NLO power corrections. The first perturbative correction to the LO term in (4) has been found in [4] and the NLO power corrections have been discussed in [5, 6] and more recently in [7] within the light-cone sum rules.

The leading momentum power dependence of the form factor (4) is dictated by the scaling property of the pion DA. But the coefficients of the power expansion depend crucially on the internal pion dynamics, which is parameterized by the nonperturbative pion DAs, $\varphi_\pi(x)$, defined at some normalization scale $\mu$, with $x$ being the fraction of the pion momentum, $p$, carried by a quark. At asymptotically large normalization scale $\mu \to \infty$ the DAs are determined in pQCD:

$$\varphi_{\pi,as}^{(2)}(x) = 6x(1-x), \quad \varphi_{\pi,as}^{(4)}(x) = 30x^2(1-x)^2. \quad (6)$$

However, for the description of the experimentally observable hard exclusive processes one needs to know the DAs normalized at virtuality $\mu^2 \sim 1$ GeV$^2$. The aim of this letter is to calculate the pion transition form factor in the kinematical region up to moderately large $Q^2$ and extract from its power expansion in $1/Q^2$ the pion DAs at normalization scale typical for hadrons. The calculations carried out within the effective model with nonlocal quark-quark interaction are consistent with the chiral anomaly and result in the relations between the DAs of twists 2 and 4 and the dynamically generated nonlocal quark mass. The usage of the covariant nonlocal low-energy model based on the Schwinger-Dyson approach to dynamics of quarks and gluons has many attractive features as the approach preserves the gauge invariance, it is consistent with the low-energy theorems and takes into account the large distance dynamics of the bound state. Furthermore, the intrinsic nonlocal structure of the model may be motivated by fundamental QCD interactions induced by the instanton and gluon exchanges.

The effective quark-pion dynamics motivated by the instanton-induced interaction[1)] may be summarized in terms of the dressed quark propagator

$$S^{-1}(p) = \widehat{p} - M\left(p^2\right),$$

the quark-pion vertex

$$\Gamma_\pi^a\left(k,p,k'=k+p\right) = \frac{i}{f_\pi} F(k^2,k'^2)\gamma_5\tau^a,$$

$$F\left(k^2,k'^2\right) = \sqrt{M\left(k^2\right)M\left(k'^2\right)},$$

and the quark-photon vertex satisfying the Ward-Takahashi identity

$$\Gamma^\mu\left(k,q,k'=k-q\right) = eQ\left[\gamma_\mu - \left(k+k'\right)_\mu G\left(k^2,k'^2\right)\right],$$

$$G\left(k^2,k'^2\right) = \frac{M\left(k'^2\right) - M\left(k^2\right)}{k'^2-k^2},$$

where $M(k^2)$ is the dynamically generated quark mass. The dynamical quark mass characterizes the momentum dependence of an order parameter for spontaneous breaking of the chiral symmetry and may be expressed in terms of the gauge invariant nonlocal quark condensate [9]. The inverse size of the nonlocality scale, $\Lambda$, is naturally related to the average virtuality of quarks that flow through the vacuum, $\lambda_q^2 \sim \Lambda^2$. The value of $\lambda_q^2$ is known from the QCD sum rule analysis, $\lambda_q^2 \approx \approx 0.4 \pm 0.1 \mathrm{GeV}^2$ [10], and, within the instanton model, may be expressed through the average instanton size, $\rho_c$, as $\lambda_q^2 \approx 2\rho_c^{-2}$ [11]. The pion weak decay constant is expressed by the Pagels-Stokar formula

$$f_\pi^2 = \frac{N_c}{4\pi^2}\int_0^\infty du \frac{uM(u)\left[M(u) - uM'(u)/2\right]}{D^2(u)}, \qquad (7)$$

where $M'(u) = \frac{d}{du}M(u)$ and $D(u) = u + M^2(u)$.

The invariant amplitude for the process $\gamma^*\gamma^* \to \pi^0$ is given by

$$A\left(\gamma^*\left(q_1,\epsilon_1\right)\gamma^*\left(q_2,\epsilon_2\right) \to \pi^0\left(p\right)\right) = = -ie^2\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu q_1^\rho q_2^\sigma M_{\pi^0}\left(q_1^2,q_2^2\right),$$

where $\epsilon_i^\mu$ are the photon polarization vectors. In the effective model one finds the contribution of the triangle diagram to the invariant amplitude as

$$A\left(\gamma_1^*\gamma_2^* \to \pi^0\right) = -ie^2\frac{N_c}{3f_\pi}\times$$

$$\times\int\frac{d^4k}{(2\pi)^4}F(k_+^2,k_-^2)\left\{\mathrm{tr}[i\gamma_5 S(k_-)\widehat{\epsilon}_2 S[k-q/2]\widehat{\epsilon}_1 S(k_+)] +\right.$$

$$+\mathrm{tr}[i\gamma_5 S(k_-)S[k-q/2]\widehat{\epsilon}_1 S(k_+)]\times$$

$$\times\left(\epsilon_2, 2k-q_1\right)G\left((k-q/2)^2,k_-^2\right) +$$

$$+\mathrm{tr}[i\gamma_5 S(k_-)\widehat{\epsilon}_2 S[k-q/2]S(k_+)]\left(\epsilon_1,2k+q_2\right)\times$$

$$\left.\times G\left(k_+^2,(k-q/2)^2\right)\right\} + \left(q_1 \leftrightarrow q_2; \epsilon_1 \leftrightarrow \epsilon_2\right), \qquad (8)$$

where $p = q_1 + q_2$, $q = q_1 - q_2$, $k_\pm = k \pm p/2$. In the adopted chiral limit $\left(p^2 = m_\pi^2 = 0\right)$ with both photons real $\left(q_i^2 = 0\right)$ one finds the result

$$M_{\pi^0}(0,0) =$$

$$\times\frac{N_c}{6\pi^2 f_\pi}\int_0^\infty du\frac{uM(u)\left[M(u) - 2uM'(u)\right]}{D^3(u)} = \frac{1}{4\pi^2 f_\pi}, \qquad (9)$$

consistent with the chiral anomaly.

The LO behavior of the form factor at large photon virtualities is given by the contribution of the first term in (8) and the NLO power corrections are generated by the second and third terms in (7) and also appear as the correction to the first term. Thus, for large $q_1^2 = q_2^2 = -Q^2/2$ and $p^2 = 0$ the form factor has the asymptotics

$$M_{\pi^0}\left(-Q^2/2,-Q^2/2\right)\big|_{Q^2\to\infty} = = \frac{4f_\pi}{3Q^2}\left(1+\frac{\Delta^2}{Q^2}\right) + O\left(\frac{1}{Q^6}\right), \qquad (10)$$

[1)]See for a review e.g. [8].

$$\Delta^2 = \frac{N_c}{4\pi^2 f_\pi^2}\int_0^\infty du \frac{u^2 M(u)(M(u)+\frac{1}{3}uM'(u))}{D^2(u)}, \quad (11)$$

which is in agreement with the expressions (4), (5) for the asymptotic coefficients at $\omega = 0$. The parameter $\Delta^2$ has an extra power of $u$ in the integral with respect to (7) and thus it is proportional to the matrix element $\left\langle \pi(p) \left| g_s \bar{d}\tilde{G}_{\alpha\mu}\gamma_\alpha p_\mu u \right| 0 \right\rangle$. The power correction (11) is the sum of the positive contribution coming from the higher Fock states in the pion, effectively taken into account by the second and third terms in (8), and also the negative two-particle contribution due to the first term in (8)[2)]. Note, that the model provides the opposite sign of the power correction comparing with the QCD sum rule prediction [5].

In general case at large $Q^2$ the model calculations reproduce the QCD factorization result (4),(5) with the DAs given by

$$\varphi_\pi^{(2)}(x) = \frac{N_c}{4\pi^2 f_\pi^2}\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi} \times$$
$$\times \int_0^\infty du \frac{F(u+i\lambda\bar{x}, u-i\lambda x)}{D(u-i\lambda x)\,D(u+i\lambda\bar{x})} \times$$
$$\times \left[xM(u+i\lambda\bar{x}) + (x \leftrightarrow \bar{x})\right], \quad (12)$$

$$\varphi_\pi^{(4)}(x) = \frac{1}{\Delta^2}\frac{N_c}{4\pi^2 f_\pi^2}\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi} \times$$
$$\times \int_0^\infty du \frac{uF(u+i\lambda\bar{x}, u-i\lambda x)}{D(u-i\lambda x)\,D(u+i\lambda\bar{x})} \times$$
$$\times \left[\bar{x}M(u+i\lambda\bar{x}) + (x \leftrightarrow \bar{x})\right]. \quad (13)$$

In these expressions the $u$-variable plays the role of the quark transverse momentum squared, $\vec{k}_\perp^{\,2}$, and $\lambda x$, $-\lambda\bar{x}$ are the longitudinal projections of the quark momentum on the light cone directions. The model DAs are defined at the normalization scale characterized by the vacuum nonlocality $\mu^2 \sim \Lambda^2$. Concerning the LO DA, $\varphi_\pi^{(2)}(x)$, the similar results within the instanton model have been earlier derived in [13, 14].

In Fig.1 the normalized by unity LO and NLO pion DAs are illustrated in comparison with perturbative asymptotic DAs. For the numerical analysis the dynamical mass profile is chosen in the Gaussian form $M(k^2) = M_q \exp(-2k^2/\Lambda^2)$, where we take $M_q = 350\,\text{MeV}$ and fix $\Lambda = 1.29\,\text{GeV}$ from the pion constant (7). Then, the value $\Delta^2 \equiv J^{(4)}(\omega=1)/J^{(2)}(\omega=1) = 0.205\,\text{GeV}^2$ is obtained which characterizes the scale of the power corrections in the hard exclusive processes. The mean square radius of the pion for the transition $\gamma^*\pi^0 \to \gamma$ is $r_{\pi\gamma}^2 = (0.566\,\text{fm})^2$ and numerically close to the value derived from (3). As it is clear from Fig. 1, the predicted pion DAs at the realistic choice of the model parameters are close to the asymptotic DAs. The corresponding conclusion with respect the LO DA is in agreement with the results obtained in [15, 16] as it is seen from comparison of Figs.1 and 2.

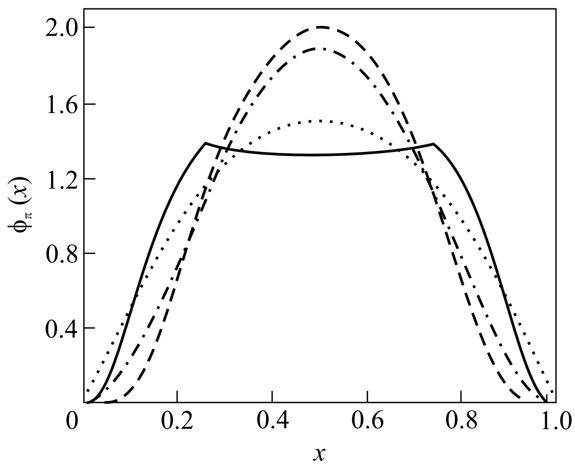


Fig.1. The pion distribution amplitudes (normalized by unity): the model predictions for twist-2 (solid line) and twist-4 (dashed line) components and the perturbative asymptotic limits of twist-2 (dotted line) and twist-4 (dash-dotted line) amplitudes

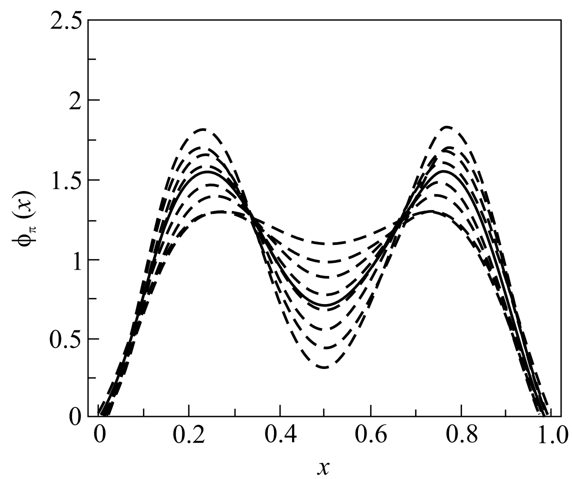


Fig.2. An admisseble set of the twist-2 pion distribution amplitudes (dashed lines, the best fit is solid line) as predicted within the QCD sum rules (from [15b]) with vacuum nonlocality parameter $\lambda_q^2 = 0.4\,\text{GeV}^2$ defined at $\mu^2 \approx 1\,\text{GeV}^2$

[2)]In [12] only part of the NLO power corrections has been discussed.

The asymptotic coefficients $J^{(2,4)}(\omega)$ given by (5), (12) and (13) may be identically rewritten in the form

$$J^{(2)}(\omega) = -\frac{1}{\pi^2 f_\pi} \int_0^\infty du u \times \tag{14}$$

$$\times \int_0^\infty dv \left\{ \frac{M^{1/2}(z_-)}{D(z_-)} \frac{\partial}{\partial z_+} \left( \frac{M^{3/2}(z_+)}{D(z_+)} \right) + (z_- \longleftrightarrow z_+) \right\},$$

$$J^{(4)}(\omega) = \frac{2}{\pi^2 f_\pi} \int_0^\infty du \times \tag{15}$$

$$\times \int_0^\infty dvv \left\{ \frac{M^{1/2}(z_-)}{D(z_-)} \times \right.$$

$$\left. \times \left[ \frac{M^{3/2}(z_+)}{D(z_+)} + u \frac{\partial}{\partial z_+} \left( \frac{M^{3/2}(z_+)}{D(z_+)} \right) \right] + (z_- \longleftrightarrow z_+) \right\},$$

where $z_\pm = u + v(1 \pm \omega)$. With the model parameters given above we find the asymptotic coefficients $J^{(2)}(\omega = 1) = 0.171\,\mathrm{GeV}$ and $J^{(4)}(1)/J^{(2)}(1) = 0.254\,\mathrm{GeV}^2$ for the process $\gamma\gamma^* \to \pi^0$. When the error in the experimental fit is taken into account, the estimate of the LO coefficient, $J^{(2)}(1)$, is in agreement with the fit of CLEO data $J^{(2)}_{\rm exp}(1) = 0.16 \pm 0.03\,\mathrm{GeV}$. The NLO power correction, $\Delta^2$, grows by 20% with changing the kinematics from equally distributed photon virtualities to asymmetric distribution.

In Figs.3 and 4 we plot the model predictions for the form factors $F_{\pi\gamma*}(Q^2) = M_{\pi^0}(-Q^2, 0)$ and $F_{\pi\gamma^*\gamma^*}(Q^2) = M_{\pi^0}(-Q^2/2, -Q^2/2)$ multiplied by square momentum $Q^2$ for the process $\gamma\gamma^* \to \pi^0$ and $\gamma^*\gamma^* \to \pi^0$, correspondingly. In Fig.3 we also indicate the CLEO data. In the model form factors the perturbative $\alpha_s-$ corrections [4] to the leading twist-2 term are taken into account with the running coupling, $\alpha_s(Q^2)$, that has zero at zero momentum [17]. With such effective behaviour in the infrared region the perturbative corrections do not influent the chiral anomaly. At high momentum squared the leading perturbative correction provides negative contribution to the form factors and compensate the NLO power corrections in the region $2-10\,\mathrm{GeV}^2$. The unknown perturbative corrections to the twist-4 contribution is considered as inessential. The power corrections generated by the twist-3 pion DAs are also negligible since they are proportional to the small current quark mass.

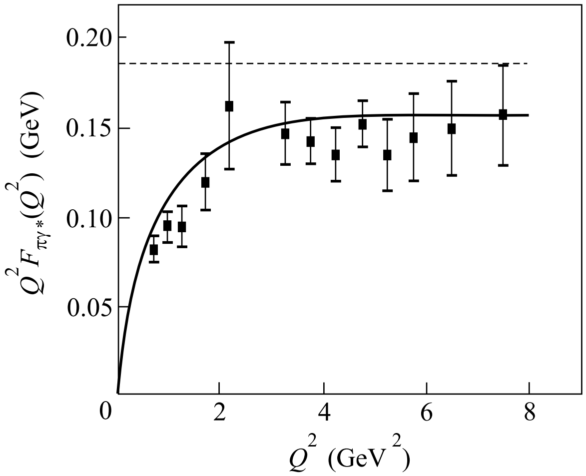


Fig.3. The pion-photon transition form factor $Q^2 F_{\pi\gamma^*}(Q^2)$ (solid line) and its perturbative limit $2f_\pi$ (dotted line). The experimental points $(Q^2 F_{\pi\gamma^*\gamma})$ are taken from [2]

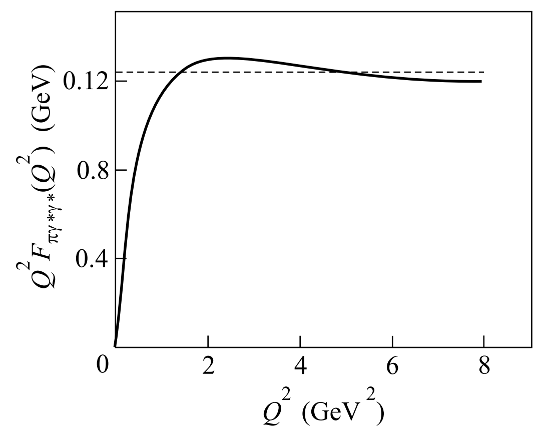


Fig.4. The pion-photon transition form factor $Q^2 F_{\pi\gamma^*\gamma^*}(Q^2)$ (solid line) and its perturbative limit $4f_\pi/3$ (dashed line)

In conclusion, within the covariant nonlocal model describing the quark-pion dynamics, we obtain the $\pi\gamma^*\gamma^*$ transition form factor in the region up to moderately high momentum transfer squared, where the perturbative QCD evolution does not reach the asymptotic regime yet. From the comparison of the kinematical dependence of the coefficients of the power expansion in $1/Q^2$ of the transition pion form factor, as it is given by pQCD and the nonperturbative model, the relations Eqs.(12), (13) between the pion DAs and the dynamical quark mass and quark-pion vertex are derived. The other possible sources of contributions to the form factor arise from inclusion into the model of the low lying vector and axial-vector mesons. They do not change the result given by the chiral anomaly (9) for the two-gamma pion decay. The contributions of the vector mesons to the leading order asymptotics of the form factor are expected to be small, but they may be more important in treating the twist-4 power corrections and the pion mean radius.

I am grateful to A. P. Bakulev, W. Broniowski, A. Di Giacomo, A. S. Gorski, N. I. Kochelev, S. V. Mikhailov, M. K. Volkov, L. Tomio and V. L. Yudichev for many useful discussions on topics related to this work. The work is supported by RFBR Grants # 01-02-16431 and # 02-02-16194 and by Grant # INTAS-2000-366.

1. CELLO Collaboration (H.-J. Behrend et. al.), Z. Phys. **C49**, 401 (1991).
2. CLEO Collaboration (J. Gronberg et. al.), Phys. Rev. **D57**, 33 (1998).
3. G. P. Lepage and S. J. Brodsky, Phys. Lett. **B87**, 359 (1979); Phys. Rev. **D22**, 2157 (1980).
4. F. Del Aguila and M. K. Chase, Nucl. Phys. **B193**, 517 (1981); E. Braaten, Phys. Rev. **D28**, 524 (1983); E. P. Kadantseva, S. V. Mikhailov, and A. V. Radyushkin, Sov. J. Nucl. Phys. **44**, 326 (1986).
5. V. L. Chernyak, A. R. Zhitnitsky, and I. R. Zhitnitsky, Sov. J. Nucl. Phys. **38**, 645 (1983); V. A. Novikov, M. A. Shifman, A. I. Vainshtein et al., Nucl. Phys. **B237**, 525 (1984).
6. A. S. Gorsky, Sov. J. Nucl. Phys. **50**, 498 (1989) [Yad. Fiz. **50**, 796 (1989)].
7. A. Khodjamirian, Eur. Phys. J. **C6** , 477 (1999); T. Yeh, hep-ph/0107018.
8. I. V. Anikin, A. E. Dorokhov, and L. Tomio, Phys. Part. and Nucl. **31**, 509 (2000).
9. A. E. Dorokhov and W. Broniowski, Phys. Rev. **D65**, 094007 (2002).
10. V. M. Belyaev and B. L. Ioffe, Sov. Phys. JETP **56**, 493 (1982) [Zh. Eksp. Teor. Fiz. **83**, 876 (1982)]; A. A. Ovchinnikov and A. A. Pivovarov, Yad. Fiz. **48**, 1135 (1988).
11. A. E. Dorokhov, S. V. Esaibegian, and S. V. Mikhailov, Phys. Rev. **D56**, 4062 (1997).
12. A. E. Dorokhov, hep-ph/0206088.
13. V. M. Braun and I. E. Filyanov, Z. Phys. **C48**, 239 (1990).
14. S. V. Esaibegyan and S. N. Tamaryan, Sov. J. Nucl. Phys. **51**, 485 (1990); A. E. Dorokhov, Nuovo Cimento **A109**, 391 (1996); V. Yu. Petrov and P. V. Pobylitsa, hep-ph/9712203.
15. S. V. Mikhailov and A. V. Radyushkin, Sov. J. Nucl. Phys. **52**, 697 (1990); A. P. Bakulev, S. V. Mikhailov, and N. G. Stefanis, Phys. Lett. **B508**, 279 (2001).
16. P. Kroll and M. Raulfs, Phys. Lett. **B387**, 848 (1996); I. V. Anikin, A. E. Dorokhov, and L. Tomio, Phys. Lett. **B475**, 361 (2000); M. Diehl, P. Kroll, and C. Vogt, Eur. Phys. J. **C22**, 439 (2001).
17. K. Van Acoleyen and H. Verschelde, hep-ph/0203211.