Dynamics of magnetic solitons in ^{3}He-B

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The production and propagation of **n**-texture solitons in the B phase of ^{3}He in a magnetic field is analyzed. It is conceivable that **n** solitons were observed in ^{3}He-B by Webb, Sager, and Wheatley[3] in their experiments after the magnetic field was turned off.

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As Maki and Kumar have shown,[1] the presence of a magnetic field in ^{3}He-B, whose condensate is described in terms of the unit vector **n** and the angle θ, leads to the formation of **n** textures that are analogous to the magnetic walls in nematics.[2] In this paper we examine the production and propagation of **n** solitons after the nonuniform magnetic field has been turned off. In a similar case Webb, Sager, and Wheatley[3] observed the propagation of slow magnetic perturbations in ^{3}He-B, whose velocity depends on the exciting magnetic field in a complicated way. To identify the magnetic perturbations with **n** solitons, which were observed in Ref. 3 and investigated in this work, we must measure the dependence of the wave's velocity on the magnetic fields in greater detail; however, the results of Ref. 3 can be explained qualitatively in terms of **n** solitons.

We examine the Lagrangian function $L = T - U$ in ^{3}He-B in a magnetic field **H**. The potential energy U is comprised of the gradient energy[4]

$$F_B = \int d\mathbf{r} \{ K_1 (\mathrm{div}\,\mathbf{n})^2 + K_2 (\mathbf{n}\,\mathrm{rot}\,\mathbf{n})^2 + K_3 (\mathbf{n}\,\mathrm{rot}\,\mathbf{n})^2 - K_4\, \mathrm{div}\,\mathbf{n}\, (\mathbf{n}\,\mathrm{rot}\,\mathbf{n}) \} \tag{1}$$

and the orientational energy in a magnetic field[4]

$$F_{\mathrm{H}} = -a \int d\mathbf{r}\, (\mathbf{n}\mathbf{H})^2, \tag{2}$$

where $K_i = (b_i/64)\hbar^2 \rho_s/m$, $b_1 = 13$, $b_2 = 11$, $b_3 = 16$, $b_4 = 13\sqrt{15}/8$, ρ_s is the superfluid density, and m is the mass of an ^{3}He atom; the expression for a is given by Eq. (6) taken from Ref. 4. Equation (1) corresponds to Leggett's configuration ($\cos\theta = -1/4$), since the dipole energy greatly exceeds F_{H} over a broad range of magnetic fields. The kinetic energy is[1]

$$T = \frac{\chi_B}{2\gamma_0^2} \int d\mathbf{r}\, [(\vec{\omega} - \vec{\omega}_0)^2 - \vec{\omega}_0^2], \tag{3}$$

where χ_B is the susceptibility of ^{3}He-B and γ_0 is the gyromagnetic ratio of ^{3}He nuclei.

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$$\vec{\omega}_o = \gamma_o \mathbf{H}\ , \ \vec{\omega} = -\frac{5}{4}\mathbf{n}\times\mathbf{n}_t + \frac{\sqrt{15}}{4}\mathbf{n}_t\ , \ \theta = \theta_o\ .$$

We can now easily obtain an equation of motion for the **n** vector. We confine ourselves to analyzing the planar textures. We assume that the magnetic field is directed along the x axis and $\mathbf{n}=\{\cos\phi(x), \sin\phi(x), 0\}$. This case, which pertains to splay-bend deformations, is described by the equation

$$\phi_{tt} - c_3^2\phi_{xx} + \frac{c_3^2}{2\xi_{H3}\phi_x}\frac{\partial}{\partial x}\left[\sin^2\phi\,(1 + a^2\xi_{H3}^2\phi_x^2)\right] = 0, \tag{4}$$

where

$$c_i^2 = \frac{b_i}{80}\frac{\gamma_o^2}{\chi_B}\frac{\hbar^2\rho_s}{m}\ , \ \xi_{Hi} = \frac{1}{H}\left(\frac{K_i}{a}\right)^{1/2}, \ a^2 = 1 - \frac{b_1}{b_3} = \frac{3}{16}.$$

Note that if $K_1 = K_3$ ($\alpha^2 = 0$), then the static solution of Eq. (4) will represent a wall that is perpendicular to the magnetic field.[2] The anisotropy ($K_1 \neq K_3$) causes the wall to become an asymmetric plane $x = 0$.

If the solution of Eq. (4) is sought in the form of a traveling wave: $\phi = \phi(x - ut)$, then we obtain the condition

$$\frac{u^2}{c_3^2} << \frac{b_1}{b_3} \approx 1. \tag{5}$$

If this condition is satisfied, then we can ignore the difference between K_1 and K_3 and solve the equation

$$\phi_{tt} - c^2\phi_{xx} + \Omega_H^2 \sin\phi\cos\phi = 0 \tag{6}$$

instead of Eq. (4). Here,

$$c_1 = c_3 = c\ , \ \Omega_H^2 = \gamma^2 H^2, \ \gamma^2 = \frac{4}{5}\frac{a\gamma_o^2}{\chi_B}.$$

We now describe the case in which the additional, nonuniform, magnetic field $\mathbf{H}_0(x)$ is turned off. We define the initial conditions for Eq. (6)

$$t = 0,\ \phi = 0,\ \phi_t = \delta\gamma_o H_o(x)\ , \tag{7}$$

where δ is a constant that is determined experimentally. After substituting $2\phi = \psi$ and converting to dimensional variables $\tau = \Omega_H t$ and $\xi = \Omega_H x/c$, we obtain a standard sine-Gordon equation,

$$\psi_{\tau\tau} - \psi_{\xi\xi} + \sin\psi = 0 \tag{8}$$

with the initial conditions

$$\tau = 0,\ \psi = 0,\ \psi_\tau = 2\omega(\xi)\ . \tag{9}$$

We select $\omega(\xi)$ in the form used by Maki and Kumar[5] for the investigation of the dynamics of solitons in ^{3}He-A by using the inverse-scattering method that was developed by Ablowitz *et al.*[6] for the sine-Gordon equation

$$\omega\ (\xi)\ =\ \omega\ \theta\ (l^2 - \xi^2\ ), \tag{10}$$

where $l = L/\xi_H$, $\omega = \delta\gamma_0 H_0/\gamma H$, $2L$ is the "length" of the pulse, and H_0 is the strength of the magnetic field that is turned off. We can use in this case the results of Ref. 5. To produce N pairs of solitons or pulsating (breather) modes,[6] the following inequality must be satisfied[5]:

$$I\ >\ I_N\ . \tag{11}$$

Here,

$$I\ =\ l\omega\ ,\quad I_N\ =\ \pi\left(N\ -\ \frac{1}{2}\right),\quad N\ =\ 1, 2, \ldots$$

The soliton pairs can be formed provided that[5]

$$\omega\ >\ \omega_N \tag{12}$$

(the inverse inequality corresponds to the pulsating (breather) modes). The dependence of ω_N on I was determined numerically[5]; however, we can easily obtain for ω_N the following analytic expression:

$$\omega_N\ =\ \frac{1 + I^2}{I\,(1\ +\ I^2 - I_N^2\,)^{1/2} - I_N}\ , \tag{13}$$

which contains a small inaccuracy only when $\omega_N \to 1$. Finally, the velocity of Nth soliton is

$$u_N\ =\ c\left(1 - \frac{\omega_N^2}{\omega^2}\right)^{1/2}. \tag{14}$$

We shall briefly consider the dependence of the velocity of a soliton on the temperature T and on the fields H_0 and H. The temperature dependence of the maximum velocity c of a soliton $\propto \rho_S^{1/2} \propto (1 - T/T_c)^{1/2}$ (here the dependence of χ_B on T is disregarded) coincides with the temperature dependence of the rate of magnetic perturbations observed in Ref. 3. This rate is approximately three to four times smaller then c; moreover, the rate of magnetic perturbations increases with decreasing magnetic field H_M and increasing H_R field.[3] According to Eq. (14), the velocity of an **n** soliton has similar dependences on the fields H and H_0, which correspond to H_M and H_R. We can easily see this when $\omega_N \sim 1$ and when the dependence on H and H_0 is determined primarily by the quantity $\omega \propto H_0/H$. We should also note that the temperature dependence of the soliton's velocity, which is given by Eq. (14), is stronger than that observed in Ref. 3 if the "experimental" temperature dependence of the quantity a is taken into account, in accordance with Ref. 4.

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