Form factor of η' meson in quantum chromodynamics

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The amplitude of the process $e^+e^- \to \eta'\gamma$ at high energies is calculated in terms of quantum chromodynamics. The result does not include arbitrary and unknown parameters.

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A considerable progress has been made during the past year in describing the exclusive processes with large momentum transfer in quantum chromodynamics. After the initial investigations (see Refs. 1-3), in which the asymptotic behavior of the electromagnetic form factor of a π meson was calculated, a number of papers were published, in which the method of the relevant calculations was refined and the appen-

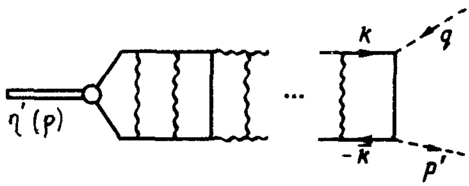

FIG. 1. The solid line represents the quark, the wavy line represents the gluon, and the dashed line denotes the γ-ray quantum.

dices covering a broad range of effects - baryon form factors, large-angle scattering of hadrons, etc. (see Refs. 4-6) - were analyzed. But all the appendices until now pertained to the processes which include a nonsinglet wave function (*WF*) of a hadron, in which the Fock gluon component is missing. In this paper we examine such mixing of gluon and quark components in the *WF* of a singlet hadronic state. We investigate the asymptotic behavior of the form factor of an η' (958) meson in the reaction $e^+e^- \to \eta'\gamma$. This problem (as applied to the decay of heavy η_c' and J/ψ mesons) was recently analyzed elsewhere.[7]

Technically, it is more convenient to investigate the amplitude with a spacelike momentum of the virtual quantum $\gamma^*(q) + \eta'(p) \to \gamma(p')$, where $q^2 = -Q^2 < 0$. A conversion to timelike quanta can be accomplished in the asymptotic form factor by a simple exchange $Q^2 \to -Q^2$. The matrix element of this process is

$$M_{\alpha\beta} = ie^2 F_{\eta'}(Q^2)\, \epsilon_{\alpha\beta\nu\mu}\, p_\nu p'_\mu, \tag{1}$$

where $Q^2 = 2pp'$ and $F_{\eta'}$ is the form factor of interest. (We assume that $p^2 = p'^2 = 0$, which is valid when $Q^2 \to \infty$ with a power-law accuracy.) Using the methods developed in Ref. 5, we can derive the following principal relation in the main logarithmic approximation:

$$iF_{\eta'}(Q^2) = \int_0^1 dx\, T(x, Q^2)\, \Phi_F(x, Q^2)\,. \tag{2}$$

Equation (2) in the physical gauge of a gluon field corresponds to taking into account only the ladder diagrams (with exact vertices and propagators) in Fig. 1. $T(x, Q^2)$ in this case corresponds to the right-hand (truncated with respect to the external ends), rigid component of the ladder (Fig. 2) that was calculated using special kinematics when $k = xp$, $\bar{k} = \bar{x}p$ $(x + \bar{x} = 1)$, and $x_{\rm eff} \sim 1$). After a suitable selection of the projector for the η' state, $T(x,Q^2)$ can be written in the form

$$T(x, Q^2) = \frac{4}{Q^2 x \bar{x}} \left(\frac{n_c}{3}\right)^{1/2} (Q_4^2 + Q_d^2 + Q_s^2), \tag{3}$$

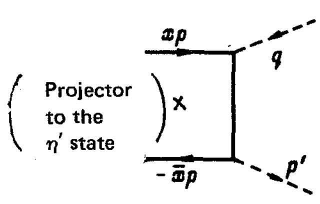

FIG. 2.

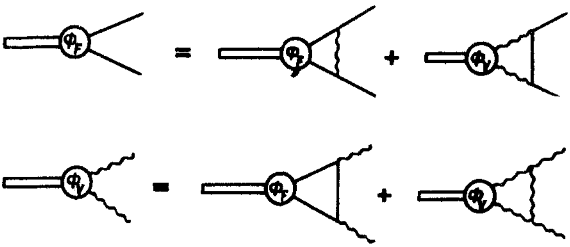

FIG. 3.

where $n_c = 3$ denotes the number of colors and $Q_{u,\ d,\ s}$ are the quark charges in e units.

The "wave function" ϕ_F corresponds to the sum of the ladder diagrams on the left-hand side of the "rigid" set in Fig. 1. Here the mixing between the quark and the gluon states is substantial, since the equation for ϕ_F "hooks on" the gluon component of the ϕ_V wave function. The corresponding equations are graphically illustrated in Fig. 3. It is convenient to seek a solution of these equations in the form of an expansion in Gegenbauer polynomials, i.e.,

$$\hat{\Phi} \equiv \begin{pmatrix} \Phi_F\,(x, Q^2) \\ \\ \Phi_V\,(x, Q^2) \end{pmatrix} = x\bar{x} \begin{pmatrix} \sum\limits_{n=0}^{\infty} f_n C_n^{3/2}\,(\bar{x} - x) \\ \\ \sum\limits_{m=1}^{\infty} g_m C_m^{5/2}\,(\bar{x} - x) \end{pmatrix}. \tag{4}$$

The asymmetry of the equations requires that n in Eq. (4) must be even and m must be odd.

In terms of f_n and g_m, the equations in Fig. 3 have the form

$$\begin{pmatrix} \dot{f}_n \\ \\ \dot{g}_{n-1} \end{pmatrix} = -\hat{\gamma}^{(n)} \begin{pmatrix} f_n \\ \\ g_{n-1} \end{pmatrix}, \quad \hat{\gamma}^{(n)} = \begin{pmatrix} \gamma_{FF}^{(n)} & \gamma_{FV}^{(n)} \\ \\ \gamma_{VF}^{(n)} & \gamma_{VV}^{(n)} \end{pmatrix}, \tag{5}$$

where $n = 0, 2, 4 \ldots$ (even) and $g_{-1} \equiv 0$. We have introduced in Eq. (5) the notation $\dot{a} \equiv \partial a(\xi)/\partial\xi$, $a = f, g$, where

$$d\xi = \frac{\alpha(Q^2)}{4\pi} \frac{dQ^2}{Q^2}$$

and $\alpha(Q^2)$ is the effective charge in quantum chromodynamics. The elements $\gamma\binom{n}{kl}$ are (n_f is the number of "flavors")

$$\gamma_{FF}^{(o)} = 0; \quad \gamma_{FF}^{(n)} = \frac{n_c^2 - 1}{2n_c} \left[4 \sum_{j=2,4,\ldots}^{n} 1/j + 1 + \frac{4n + 6}{(n+1)(n+2)} \right],$$

$$n \geq 2,$$

$$\gamma_{FV}^{(n)} = \left(\frac{n_f (n_c^2 - 1)}{2n_c} \right)^{1/2} \frac{1}{3} \frac{n(n+3)}{(n+1)(n+2)},$$

$$\gamma_{VF}^{(n)} = \left(\frac{n_f (n_c^2 - 1)}{2n_c} \right)^{1/2} \frac{12}{(n+1)(n+2)},$$

$$\gamma_{VV}^{(n)} = n_c \left[4 \sum_{j=2,4\ldots}^{n} 1/j + 1/3 + \frac{4n}{(n+1)(n+2)} + \frac{2n_f}{3n_c} \right], n \geq 2. \tag{6}$$

The positive eigenvalues of the $\hat{\gamma}$ matrix increase with increasing n; therefore, when $\xi \sim \ln \ln Q^2 \to \infty$

$$\hat{\Phi}(x, Q^2) \to x\bar{x} \begin{pmatrix} f_0 \\ 0 \end{pmatrix}, \tag{7}$$

where the constant f_0 is equal to

$$\lim_{Q^2 \to \infty} 6 \int \Phi_F(x, Q^2) dx.$$

Accordingly,

$$iF_{\eta'}(Q^2) = \frac{4}{Q^2} \sqrt{\frac{n_c}{3}} (Q_u^2 + Q_d^2 + Q_s^2) f_0. \tag{8}$$

We can express f_0, without any additional assumptions, in terms of the Bethe-Salpeter amplitude $\eta' \to$ quark + antiquark at the origin

$$2 \sqrt{\frac{n_c}{3}} f_0 p_\mu = \langle 0 | \bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d + \bar{s} \gamma_\mu \gamma_5 s | \eta'(p) \rangle, \tag{9}$$

where u, d, and s are quark-field operators. (We suggest summation over the color indices on the right-hand side of Eq. (9).) The matrix element in Eq. (9), which can be estimated in terms of the U(3) symmetry, is equal to $i\sqrt{6} p_\mu F_\pi$, where $F_\pi = 93$ meV is the decay constant $\pi \to e\nu$. A more complex estimate in Ref. 8 gives a value of ~ 0.5 $(i\sqrt{6} p_\mu F_\pi)$ for this matrix element. Using the last value, we have

$$f_0 = (0,5) \frac{3i}{\sqrt{2n_c}} F_\pi. \tag{10}$$

Equations (8) and (10) resolve this problem. Thus, the mixing of quarks and gluons in the wave function of the hadron drops out in the main term of the asymptotic form factor and the result can be expressed in terms of the observed low-energy parameters.

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