Multifrequency precession of the neutron spin in a uniform magnetic field

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It is shown that the spin of neutrons that pass through a nonmagnetic crystal in a constant, uniform field processes at several frequencies.

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Suppose that a neutron beam enters a region occupied by a constant, uniform magnetic field. As the beam advances into this region, the neutron spin rotates by an angle

$$\theta = \Omega \frac{l}{v} = \frac{2\mu H}{\hbar v} l, \qquad (1)$$

where $\Omega = 2\mu H/\hbar v$ is the Larmor spin precession frequency in a magnetic field with an intensity H, v is the neutron velocity, l is the path traversed by a particle in a magnetic field, and μ is the magnetic moment of a neutron.

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We place a nonmagnetic unpolarized crystal, say, a silicon crystal, in a magnetic field. Since such a crystal does not influence the neutron spin, at first glance the spin rotation does not change and the spin rotation angle is determined by Eq. (1), as before. Suppose that the crystal is oriented in such a way that the neutrons are diffracted in it. The diffraction essentially depends on the wavelength of incident neutrons. We recall that the neutrons are refracted as a result of entering a magnetic field. The refractive index of a neutron with a spin in the direction parallel to the magnetic field does not equal the refractive index of a neutron with a spin in the opposite direction. As a result, the magnetic field can sharply change the diffraction of a particle in the crystal.[1] If, for example, the neutron with a spin in the direction of the field is diffracted, then the neutron with opposite spin direction may not be diffracted. As a result, the phase velocities of neutrons with different spin orientations relative to the magnetic field differ sharply and depend on the single-crystal orientation. We can expect, therefore, that the spin precession will vary sharply even in a nonmagnetic crystal as a result of diffraction.

According to Ref. 2, the system of equations, which describes the dynamic diffraction in an arbitrary, magnetically ordered crystal with polarized nuclei, has the form

$$\left(\frac{k^2}{k_0^2} - 1\right)\phi(\mathbf{k}) - \sum_{\tau} \hat{g}(\vec{\tau})\phi(\mathbf{k} - 2\pi\vec{\tau}) = 0, \tag{2}$$

$$\hat{g}(\vec{\tau}) = \frac{4\pi}{V k_0^2} \sum_j (\hat{f}_{j\,\mathrm{nucl.}}(\vec{\tau}) + \hat{f}_{j\,\mathrm{magn.}}(\vec{\tau})\, e^{-i 2\pi\vec{\tau}\mathbf{r}_j} \tag{3}$$

At $\tau \neq 0$ the amplitude of the coherent magnetic scattering has the form

$$\hat{f}_{j\,\mathrm{magn.}}(\vec{\tau}) = -4\pi\mu \left[\frac{(\vec{\sigma}\vec{\tau})(\vec{\tau}\vec{\mu}_j)}{\tau^2} - \vec{\sigma}\vec{\mu}_j\right] F_j(\vec{\tau})\, e^{-w_j(\vec{\tau})}. \tag{4}$$

At $\tau = 0$ the magnetic contribution to $\hat{g}(0)$ has the form

$$\hat{g}_{\mathrm{magn.}}(0) = \frac{2m\mu}{\hbar^2 k_0^2}\vec{\sigma}\vec{\beta}, \tag{5}$$

where **B** is the macroscopic magnetic field in the target. For a nonmagnetic, nonpolarized crystal situated in a constant magnetic field **H** the structure amplitudes (3) have the form

$$\hat{g}(0) = \frac{4\pi}{V k_0^2}\sum_j f_{j\,\mathrm{nucl.}}(0) + \frac{2m\mu}{\hbar^2 k_0^2}\vec{\sigma}\mathbf{H}, \tag{6}$$

$$\hat{g}_{\tau \neq 0}(\vec{\tau}) = g(\vec{\tau}) = \frac{4\pi}{V k_0^2}\sum_j f_{j\,\mathrm{nucl.}}(\vec{\tau})\, e^{-i 2\pi\vec{\tau}\mathbf{r}_j} \tag{7}$$

We selected a quantization axis parallel to the direction of the field **H**. As a result, we can reduce the operator system (2) to two independent systems of equations for each component of the neutron spin, which are parallel (ϕ_+) and antiparallel (ϕ_-) to the quantization axis

$$\left(\frac{k^2}{k_0^2} - 1\right)\phi_\pm(\mathbf{k}) - \sum_{\vec{\tau}} g_\pm(\vec{\tau})\,\phi_\pm(\mathbf{k} - 2\pi\vec{\tau}) = 0,$$

$$g_\pm(0) = g_{\text{nucl.}}(0) \pm \frac{2m\mu}{\hbar^2 k_0^2} H; \quad g_\pm(\vec{\tau}) = g_{\text{nucl.}}(\vec{\tau}). \tag{8}$$

The solution of Eq. (8) for the neutron wave function at the exit surface of the crystal is given in Ref. 2. Using this function, we can determine the neutron-polarization vectors p_x and p_y directly, for example, for a diffracted wave (the z axis of the coordinate system is directed along the quantization axis)

$$p_x = \frac{\beta^2}{4}\left|\frac{g^2_{\text{nucl}}(\vec{\tau})}{(\epsilon_2^+ - \epsilon_1^+)(\epsilon_2^- - \epsilon_1^p)}\right|$$

$$\times\left\{\cos\left(k_0 \operatorname{Re}(\epsilon_1^+ - \epsilon_1^-)\frac{l}{\gamma_0} + \delta\right) e^{-k_0 \operatorname{Im}(\epsilon_1^+ + \epsilon_1^-) l/\gamma_0}\right.$$

$$- \cos\left(k_0 \operatorname{Re}(\epsilon_1^+ - \epsilon_2^-)\frac{l}{\gamma_0} + \delta\right) e^{-k_0 \operatorname{Im}(\epsilon_1^+ + \epsilon_2^-)\frac{l}{\gamma_0}}$$

$$- \cos\left(k_0 \operatorname{Re}(\epsilon_2^+ - \epsilon_1^-)\frac{l}{\gamma_0} + \delta\right) e^{-k_0 \operatorname{Im}(\epsilon_2^+ + \epsilon_1^-)\frac{l}{\gamma_0}}$$

$$\left. + \cos\left(k_0 \operatorname{Re}(\epsilon_2^+ - \epsilon_2^-)\frac{l}{\gamma_0} + \delta\right) e^{-k_0 \operatorname{Im}(\epsilon_2^+ + \epsilon_2^-)\frac{l}{\gamma_0}}\right\}, \tag{9}$$

P_y is obtained by substituting –sin for cos.

The differences in the values of ϵ in Eq. (9) have the form

$$\epsilon_1^+ - \epsilon_1^- = \frac{m\mu}{\hbar^2 k_0^2} H(1 + \beta) + \frac{1}{4}(A_+ - A_-),$$

$$\epsilon_1^+ - \epsilon_2^- = \frac{m\mu}{\hbar^2 k_0^2} H(1 + \beta) + \frac{1}{4}(A_+ + A_-),$$

$$\epsilon_2^+ - \epsilon_1^- = \frac{m\mu}{\hbar^2 k_o^2} H(1+\beta) - \frac{1}{4}(A_+ + A_-),$$

$$\epsilon_2^+ - \epsilon_2^- = \frac{m\mu}{\hbar^2 k_o^2} H(1+\beta) - \frac{1}{4}(A_+ - A_-),$$

$$A_\pm = \{[g_\pm(0)(1-\beta) + \beta\alpha]^2 + 4\beta g(\vec{\tau}) g(-\vec{\tau})\}^{1/2}. \qquad (10)$$

The value of A_+ is equal to A_- in the symmetrical Laue case when $\beta = 1$. The neutron polarization vector beats with the variation of H at a certain frequency which is determined by the Larmor frequency of spin precession in a magnetic field, i.e., the spin rotation angle varies according to the law (1) as a result of variation of H.

If, however, we are concerned with an asymmetric Laue diffraction ($\beta \neq 1$), then the situation will change drastically. In this case $A_+ \neq A_-$ and the neutron polarization vector beats with the variation of H at four different frequencies. We can see from Eqs. (9) and (10) that at relatively small magnetic fields with an intensity of $10^3 - 10^4$ G the multifrequency precession should be clearly observed even for thickness of crystals of the order of 10^{-3}–10^{-2} cm.

In conclusion, we note that the imaginary parts of $\epsilon_{1,2}^+$ also depend on H in the case under consideration. In other words, the magnetic field sharply changes the anomalous transmission of neutrons through a crystal even in the case of diffraction in a nonmagnetic crystal, i.e., it sharply changes the rate of neutron-induced nuclear reactions.

[1]V. G. Baryshevskii, Dokl. Akad. Nauk Belorussian SSR 23, 438 (1979).
[2]V. G. Baryshevskii, Zh. Eksp. Teor. Fiz. 70, 430 (1976) [Sov. Phys. JETP 43, 222 (1976)].

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