Thermal conductivity of ions in an undulating magnetic field at low collision frequency

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The thermal conductivity of ions captured in local magnetic mirrors ceases to increase with decreasing collision frequency ν_i at temperature 1–5 keV and decreases proportionally to ν_i at higher temperature.

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One of the mechanisms of heat loss due to an undulating magnetic field in a tokamak is the drift of particles that are captured in local magnetic mirrors between the longitudinal field coils. Until now, the thermal conductivity of locally captured particles has been considered to be inversely proportional to the collision frequency ν_i:$x_i^r \sim \delta^{3/2} v_d^2/\nu_i$ (δ is the undulating depth, v_d is the velocity of the toroidal drift, and ν_i is the frequency of Coulomb collisions) down to very low frequencies $v_i \sim \delta v_d/a$ (a is the minor radius of the torus) at which the effective time $\tau_{\text{eff}} \sim \delta/v_i$ between the collisions of captured particles is comparable to the drift time $\tau_d \sim a/v_d$ across the device.[1] The numerical modulation[2] showed, however, that a deviation from the dependence $\chi_i^r \sim \nu_i^{-1}$ and the transition to $\chi_i^r \sim \nu_i$ occur much sooner in the region of parameters that will be attained in future thermonuclear fusion devices. We show that the thermal conductivity in the region of weak collisions is much lower than $\nu_i a^2$—a value predicted by Stringer[1]—and that the frequency of the transition to this regime, which is, in fact, a factor of 10-100 greater than $\delta v_d/a$, corresponds to the temperature 1–5 keV.

The particles captured locally drift in the vertical direction at a low collision frequency without colliding. However, since the local magnetic mirrors exist in real systems only when $\theta < \theta_0$ (see Fig. 1), these particles do not leave the device but rather enter a region in which there are no magnetic mirrors. Returning along a line of

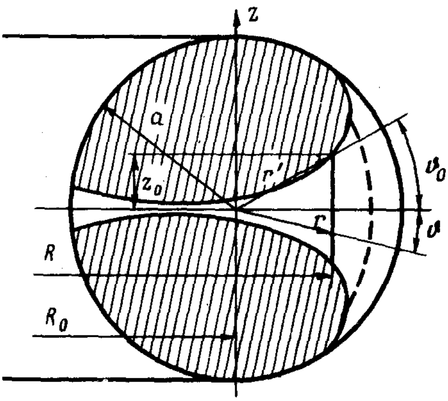

FIG. 1. Cross section of the tokamak in the vertical plane. The crosshatched area represents the region in which there are no local magnetic mirrors. The solid vertical line represents the trajectory of a locally captured particle without a radial electric field and the dashed line denotes the trajectory produced as a result of combined action of the toroidal and electric drifts.

force, they describe closed trajectories with a shift $\Delta_r \sim r\theta_0^2$ from the magnetic surface. Taking the weak collisions into account, we shall estimate the coefficient of thermal conductivity $\chi_i^r \sim f\Delta r^2 \nu_{\text{eff}} \sim \nu_i r^2 \theta_0^5/\delta^{1/2}$, where $f \sim \delta^{1/2}\theta_0$ is the fraction of locally captured particles on the magnetic surface and $\nu_{\text{eff}} = \tau_{\text{eff}}^{-1}$. This estimate is correct if the particles drift across the band in which the magnetic mirrors exist $z_0 \sim r\theta_0$ in a time that is less than τ_{eff} or at $\nu_i \lesssim \delta v_d / r\theta_0$.

In the quantitative analysis we shall use a kinetic equation that is averaged over the fast oscillations in a local magentic mirror. The equation for the correction for the distribution function ($F = F_M + f$, where F_M is a Maxwell function) in the variables $\mu = v_\perp^2/2B$ and $\epsilon = v^2/2$ has the form[3]

$$\frac{1}{\omega R} \frac{\partial(\mu J)}{\partial \mu} \frac{\partial(f + F_M)}{\partial t} = \nu_i(\epsilon) \frac{\partial}{\partial \mu}\left(\frac{\mu J}{B} \frac{\partial f}{\partial \mu}\right), \tag{1}$$

where ω is the cyclotron frequency and R is the major radius of the torus,

$$J = \int_{\phi_1}^{\phi_2} v_{\|}\, d\phi ,$$

where ϕ_1 and ϕ_2 are the reflection points ($v_{\|} = 0$) of the captured particles. The collision term on the right-hand side of this equation describes only the isotropization of the particles. Equation (1) can be solved by expanding it over the minor parameter, which is proportional to the collision frequency in the region in which there are captured particles

$$-z_0(R) < z < z_0(R),\ 1 - \Delta(z, R) < \mu B/\epsilon < 1, \tag{2}$$

where $\Delta(z,R)$ is the depth of the magnetic well. For simplicity, we assume that the z dependence of the well depth is quadratic $\Delta(z,R) = 2\delta(r)\,[1 - z^2/z_o^2(R)]$ and, limiting ourselves to the case $z_0(R) \ll r$, we determine the distribution function of the locally captured particles with the required accuracy

$$f \approx f_0 + f_1 = \frac{\partial F_M}{\partial r} \frac{z_0^2}{2r}\left[1 - \frac{z^2}{z_0^2} - \frac{1}{2\delta(r)}\left(1 - \frac{\mu B}{\epsilon}\right)\right]$$

$$+ \nu(\epsilon) \frac{\omega R z z_0^2}{4r\delta(r)\epsilon} \frac{\partial F_M}{\partial r}. \tag{3}$$

To obtain the thermal flux, we integrate the obtained distribution over the region bounded by the inequalities (2) with the weight $m\epsilon\, v_d \sin\theta$ ($v_d = v_\perp^2/R\omega$) and average over the magnetic surface. The f_0 function does not contribute to the heat flux and the contribution form f_1 has the form

$$Q_i = \frac{1}{64\sqrt{\pi}} \frac{\theta_0^5\, r^2}{\delta^{1/2}}\, \nu_i \left[0.59\left(\frac{1}{n}\frac{\partial n}{\partial r} + \frac{eE_r}{T_i}\right) - 0.148\frac{1}{T_i}\frac{\partial T_i}{\partial r}\right] nT_i. \tag{4}$$

The radial electric field in this expression is determined from the diffusion-ambipolarity condition. If we assume that the main contribution to the diffusion comes from the locally captured particles, where the electrons are still in the strong-collision regime ($\nu_e < v_d\delta/r\theta_0$) when the electron flux Γ_e is much smaller than Γ_i, the ambipolarity condition $\Gamma_i = \Gamma_e$ reduces almost to $\Gamma_i = 0$. The thermal conductivity of ions in this case is

$$\chi^r_i = 4.2 \times 10^{-3} \nu_i \theta^5_0 r^2 / \delta^{1/2}. \tag{5}$$

We shall estimate the collision frequency at which there is a transition from the "collisional" mode ($\chi^r_i \sim \nu_i^{-1}$) to the "collisionless" mode ($\chi^r_i \sim \nu_i$). To do this, we shall equate the thermal conductivities in these limiting cases,

$$0.9\, \delta^{3/2} v^2_d\, \theta^3_0 / \nu^0_i = 4.2 \cdot 10^{-3} \nu^0_i\, \theta^5_0 r^2 / \delta^{1/2}. \tag{6}$$

The left-hand side of the equation represents the thermal conductivity determined in Ref. 3 for the case $\epsilon/Nq\delta \gg 1$, where $\theta_0 = Nq\delta/\epsilon$ (N is the number of coils for the longitudinal field and $\epsilon = r/R$). The threshold frequency

$$\nu^0_i \approx 15\, \delta v_d / r\theta_0 \approx 15 v_d / NqR \tag{7}$$

is one or two orders of magnitude higher than the value predicted by Stringer.[1] At the same time, the estimate (7) is in good agreement with the threshold frequencies obtained by a numerical calculation mentioned above.[2] This agreement allows us to assume that a decrease of the thermal conductivity coefficient of the captured particles at low collision frequencies, which was observed in Ref. 2, is accounted for by the same effect—the yield of particles from the band containing local magnetic mirrors due to the vertical drift.

The drift trajectories of the captured particles, however, are not vertical lines. The radial electric field $E_r \sim T/ea$, which is always present in the plasma, forces them to rotate about the minor axis of the torus with a velocity $v_E = cE_r/B \sim v_d kR/a$, in addition to drifting vertically. The trajectories of the locally captured particles in this case are arcs of circles whose center is displaced by rv_d/v_E from the axis of the device. As a result, there displacement relative to the magnetic surface decreases by a factor of $(v_E + v_d)/v_d \sim R/a$ as compared with the previous estimate. Accordingly, the thermal conductivity coefficient (5) decreases by a factor of $(v_E + v_d)^2/v^2_d \sim R^2/a^2$. Note that χ^r_i, in fact, cannot be sharply reduced in the regions in which $v_E \gg v_d R/a$. The captured particles move along the equipotential surfaces of the electric field in the regions in which $v_E \gg v_d R/a$. But these surfaces coincide with the magnetic surfaces to within a/R. Therefore, the radial displacement of particles even in a very large electric field is equal to $r\theta^2_0 a/R$. Thus, the thermal conductivity coefficient of the locally captured particles in real electric fields can be estimated as follows:

$$\chi^r_i \approx 4 \times 10^{-3} \nu_i\, \theta^5_0 r^2 a^2 / R^2 \delta^{1/2}, \tag{8}$$

and the frequency at which the transition to this regime occurs is

$$\nu^0_i \approx 15\, \delta v_d R / ar\, \theta_0 \approx 15 v_d / Nqa. \tag{9}$$

The collisionless diffusion regime is realized at the temperatures

$$T > (10^{-16}\, nBRNqa)^{2/5}, \tag{10}$$

where T, n, B, R, and a are measured in eV, cm^{-3}, Oe, cm, and cm, respectively. This temrperature is equal to 1 keV for the modern devices and to ~ 5 keV for the devices with the reactor parameters.

In conclusion, we shall estimate the thermal conductivity of captured particles (8). We determine the angle θ_0 at which χ_i^r is equal to the neoclassical thermal conductivity

$$\theta_0 \approx 10\,(150\, T/\epsilon B^2 a^2 N^{1/2} q^{1/2})^{2/9}.$$

The value of θ_0 for most devices is equal to $0.2 - 0.3$. Therefore, when $\theta_0 < 0.2$ the thermal conductivity of locally captured particles is negligible at any collision frequencies.

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