Thermoelectric effects in distributed Josephson S-N-S junctions

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The effects in a distributed Josephson S-N-S junction, which occur when its electrodes are at different temperatures, are examined theoretically. It is shown that this is equivalent to the case in which all parts of the junction have the same temperature, but in which flows a uniformly distributed external current.

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Thermoelectric effects in lumped S-N-S junctions were investigated by Aronov and Gal'perin.[1]

We investigate in this paper the thermoelectric effects that can be observed in extended S-N-S Josephson junctions. We shall show their strong dependence on the applied external magnetic field.

We shall investigate an S-N-S junction that has cross-sectional dimensions l and w ($l > w$, $w \leqslant \lambda_J$) and a thickness d of the normal interlayer. Here λ_J is the Josephson penetration depth of the magnetic field. We assume that the shores of the junction, i. e., the superconducting electrodes, have a different temperature -- T_1 and T_2, respectively.

The equation for the phase difference ϕ of the wave function of the shores is obtained by an elementary extension of the results of Ref. 1

$$\frac{\Phi_0}{2\pi L}\frac{\partial^2\phi}{\partial x^2} - \frac{\alpha(T_1 - T_2)}{R_n} - \frac{\hbar}{2eR_n}\frac{\partial\phi}{\partial t} = j_c \sin\phi . \tag{1}$$

Here Φ_0 is a magnetic-flux quantum, L is the specific inductance of the shores, i. e., the inductance of the superconducting strips per unit area (in units of Henry), α is the thermoelectromotive force of the normal interlayer ($\sim 10^{-6}$ V/K), R_n is the resistance of the normal interlayer per unit area (in ohm·m^2), and j_c is the critical current density of the junction; the X axis is directed along the long side l of the junction. The specific inductance L and the Josephson penetration depth λ_J are related by the simple relation

$$L = \frac{\Phi_0}{2\pi j_c \lambda_J^2} .$$

Equation (1) is readily transformed to

$$\frac{\partial^2\phi}{\partial x^2} - \eta\frac{\partial\phi}{\partial t} = \frac{1}{\lambda_J^2}\sin\phi - \frac{1}{\lambda_J^2}\frac{j_e}{j_c} , \tag{2}$$

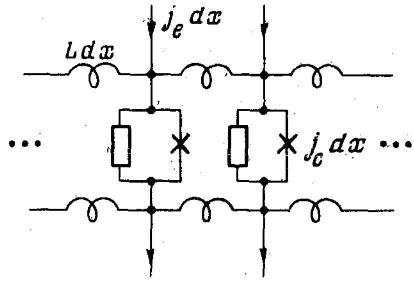

FIG. 1. Equivalent circuit of *S-N-S* junction whose shores have a different temperature.

where we have introduced the notations $\eta = L/R$ and $j_e = -\alpha(T_1 - T_2)/R_n$. It follows from Eq. (2) that the processes occurring in our distributed junction with shores at different temperatures must be completely analogous to those in such a junction with an identical temperature for the shores but with an external current j_e introduced uniformly into it. The equivalent circuit of such a junction is shown in Fig. 1. This means that under certain conditions the Josephson eddies can appear in an extended Josephson junction, whose shores are maintained at different temperatures. These eddies, which are set in motion because of the temperature difference (equivalent to an external current j_e) along the junction, produce a high-frequency voltage between the shores of the junction. This voltage which has a dc component proportional to the varying component (non-steady-state Josephson effect), is strongly dependent on the external magnetic field.

We must emphasize that all these effects occur in a junction that is not connected to an external electric circuit. Only a difference in the temperatures of the junction shores is necessary for these effects to occur.

We examine some consequences of this statement.

1. Assume that $l << \lambda_J$ and that the junction is in an external magnetic field parallel to the plane of the junction and perpendicular to the edge l. Then, the total maximum nondissipative current through the junction is determined by the known[2] relation

$$I_{max} = j_e\, l w \left| \frac{\sin kl/2}{kl/2} \right|, \quad k = 2\pi B_e d/\Phi_0, \tag{3}$$

where B_e is the induction of the external magnetic field. In our case this formula gives the critical temperature difference at which a Josephson generation of electromagnetic oscillations occurs in the junction

$$(T_1 - T_2)_c = R_n \frac{I_{max}}{\alpha l w}.$$

In particular, it follows from Eq. (3) that the oscillation begins at any nonzero temperature difference each time the junction has an integral number of magnetic-flux quanta. The potential difference between the shores that occurs at

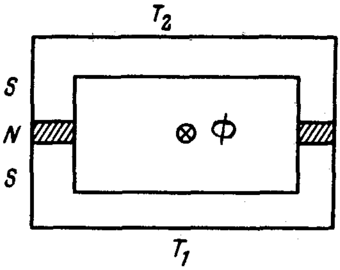

FIG. 2. Two-junction *S-N-S* loop for observing the thermoelectric analog of the Mercereau effect.

$T_1 - T_2 > (T_1 - T_2)_e$ has a component which depends strongly and nonmonotonically on B_e.

2. If $l \gg \lambda_J$, then the condition for the onset of the non-steady state and of the motion of the eddies can be obtained by using the result of Ref. 3, in which the dependence of the critical current of a long junction with a uniform external current on the external magnetic field was calculated.

3. Finally, we shall consider the thermoelectric analog of the Mercereau effect.[4] If two identical lumped *S-N-S* contacts are connected into a closed superconducting loop (Fig. 2), and the upper and lower halves of the loop are maintained at different temperatures, then the critical temperature difference ΔT_c, for which a potential difference appears between these halves, will be a nonmonotonic function of the magnetic flux Φ that is enclosed within this superconducting loop

$$\Delta T_c = \frac{2j_c R_n}{\alpha} \cos \pi\Phi/\Phi_0 .$$

Simple estimates show that if the *S-N-S* junction has $j_c \sim 10^2$ A/m^2, $R_n \sim 10^{-14}$ ohm·m^2, and $\alpha \sim 10^{-6}$ V/K, then the effects discussed in this paper must occur at a maximum temperature difference of $\sim 10^{-6}$ K at the shores of the junction. The typical junction parameters given here can be obtained in an *S-N-S* junction with a thickness $d \sim 10^{-3}$ cm of the normal layer.

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