Two-dimensional electrons in a strong magnetic field

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The wave functions and spectra of complexes comprised of two or more electrons are determined for a two-dimensional case in the strong-field limit (the magnetic length is shorter than the effective Bohr radius). The electron complexes, which are localized in the charged impurity, are calculated. The spectra for the exciton and spin-wave excitations are obtained for a close-packed electron system (all the states of the lower Landau level are occupied).

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We present several quantum-mechanical results pertaining to the behavior of a two-dimensional electron system in a strong magnetic field. Lately, such a system has been intensively investigated mainly in connection with the two-dimensional Wigner crystal.[1]

1. We shall write the energy operator of the electron system in the gauge

$$A_x = -\frac{1}{2}Hy, \quad A_y = \frac{1}{2}Hx: \quad \hat{H} = \hat{H}_0 + \hat{V},$$

$$\hat{H}_0 = \frac{1}{2m^*}\sum_i \left\{ \hat{\mathrm{p}}_i^2 + \frac{\hbar}{l^2}\left(p_{ix}y_i - p_{iy}x_i\right) + \frac{\hbar^2}{4l^2} r_i^2 \right\} + \mu^* H \sum_i \sigma_{iz} \tag{1}$$

$$\hat{V} = \frac{1}{2}\sum_{i,k} V(\mathrm{r}_i - \mathrm{r}_k), \quad l^{-2} = eH/\hbar c$$

where μ^* is an effective magneton, which can be substantially greater than the Bohr magneton for a superconductor. We shall not concretely define the interaction now

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but assume that it is isotropic, i.e., it depends on r^2. We shall also assume that the magnetic field is strong, so that the matrix elements of the potential, which transfer the electrons from one Landau level to another, can be disregarded. If $V(r) = e^2/\epsilon r$ (ϵ is the dielectric constant), then this requirement means that $l \ll \hbar^2\epsilon/m^*e^2$. We shall attempt to diagonalize the remaining operator exactly. This operator conserves the particles in each level. It is clear that $\hat{H}_0$ does not change as a result of an orthogonal transformation of the r_i variables, which preserves the form $\Sigma_i r_i^2$ and $\Sigma_i p_{ix} y_i$. Therefore, the center-of-mass motion can be separated in a system of like particles (in contrast with particles having unlike charges) and we can use, for example, the variables

$$\mathbf{R} = \frac{\sum_i \mathbf{r}_i}{\sqrt{N}}, \quad \vec{\rho}_1 = \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}}, \quad \vec{\rho}_2 = \frac{\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3}{\sqrt{6}}, \ldots$$

$$\vec{\rho}_{N-1} = \frac{\mathbf{r}_1 + \mathbf{r}_2 \ldots - (N-1)\mathbf{r}_N}{\sqrt{N(N-1)}}. \tag{2}$$

Since the interaction depends on the difference in coordinates only, the wave function of the system can be written in the form $\Psi = \Psi_n^{(m)}(\mathbf{R})\Psi_{\{\vec{\rho}i\}}$, where

$$\Psi_n^{(m)}(\mathbf{r}) = Br^{|m|} e^{im\phi} L_n^{(m)}\left(\frac{r^2}{2l^2}\right)\exp\left(-\frac{r^2}{4l^2}\right) \tag{3}$$

is the solution of a two-dimensional Schrödinger equation for a particle in a magnetic field in the accepted gauge, which corresponds to the energy $\epsilon_n^{(m)} = e\hbar H/m^*c(n + |m| - m + 1/2)$, and $L_n^{(m)}$ is the Laguerre polynomial.

2. We shall analyze a two-electron system. $\Psi = \Psi_n^{(m)}(\mathbf{R})\Psi_{n_1}^{(m_1)}(\vec{\rho}_1)$ in the absence of interaction. $\hat{V}$ is diagonal in m_1 because of the isotropy, and the nondiagonal elements in n_1 are ignored. We shall, therefore, obtain a two-particle spectrum by calculating the diagonal elements. For the lower Landau level ($n = n_1 = 0$)

$$E_{m_1} = \left(\frac{e\hbar}{m^*c} - 2\mu^*\right)H + \epsilon_{m_1}, \quad \epsilon_{m_1} = \frac{1}{2^{m_1} m_1!}\int_0^\infty x^{2m_1+1} e^{-x^2/2} V(\sqrt{2}xl)\,dx. \tag{4}$$

If $V = e^2/\epsilon r$, then $\epsilon_{m_1} = e^2\Gamma(m_1 + 1/2)/2\epsilon l m_1!$. For $m_1 \gg 1$ $\epsilon_{m_1} \sim m_1^{-1/2}$. In spite of repulsion, the spectrum is discrete and it condenses toward the lower level.

3. We shall limit ourselves for three particles to the spectrum near the main Landau level. It is clear from the preceding example that the wave function of the relative motion is a product of $\exp(-\rho_1^2 + \rho_2^2/4l^2$ and the polynomial relative to the complex variables $z_k = r_k e^{i\phi k}$ (k = 1,2,3), which depends only on $z_i - z_k$ and is completely antisymmetric. We have a third-degree polynomial, a fifth-degree polynomial and a seventh-degree polynomial

$$P_3 = (z_1 - z_2)(z_1 - z_3)(z_2 - z_3), \quad P_5 = P_3 S, \quad P_7 = P_3 S^2, \quad S = \sum_{i,k} (z_i - z_k)^2, \tag{5}$$

which determine the wave functions of the three upper states whose energies are, respectively (in $e^2/\epsilon l$ units),

$$E_3 = 1.22; \qquad E_5 = 1.05; \qquad E_7 = 0.94.$$

There is more than one polynomial of required symmetry beginning with the ninth-degree polynomial. To determine the spectrum, we must solve the secular problem, and the wave functions, in contrast to Eq. (5), are dependent on $V(r)$. We shall limit ourselves to the complete orthogonal basis of the antisymmetric polynomials of $(2m + 1)$ – degree of orthogonality[6]

$$P^{(k)}_{2m+1} = \cdot\, v_1^{m_1} v_2^{m_2} + v_1^{m_2} v_2^{m_1}, \quad v_{1,2} = z_1 - z_2 \pm \frac{i}{\sqrt{3}} (z_1 + z_2 - 2z_3)$$

$$m_{1,2} = \frac{1}{2}(2m + 1 \pm 3k) \tag{6}$$

where k is an odd number and $3k \leqslant 2m + 1$.

The wave function of the densest and the highest state N of the particles is determined by the polynomial $\prod_{i<k} (z_i - z_k)$, which coincides with the Vandermondes determinant of order N. As $N \to \infty$ the particle density in this state tends to $(2\pi l^2)^{-1}$; therefore, it corresponds to a completely occupied Landau level in the limit of large N.

4. The same procedure makes it possible to solve the problem of the electronic complexes near the positively charged impurity with the potential $V(r) = -e^2/\epsilon r$. The single-electron problem has already been analyzed.[3] The correct function in this case is $\Psi_n^{(m)}(\mathbf{r})$. The simplest two-electron functions are the product of $\exp(-r_1^2 + r_2^2/4l^2)$ and the polynomials $z_1 - z_2$ and $z_1^2 - z_2^2$. The energies of these states, measured from the zero Landau level, are equal to $-1.44\, e^2/\epsilon l$ and $-1.29\, e^2/\epsilon l$, respectively. The three-electron state with the polynomial P_3 in Eq. (5) also has a negative energy, $-0.93\, e^2/\epsilon l$. In case of necessity, even more complex electronic configurations can be calculated without much difficulty. The many-electron states bound in the impurity must be taken into account in the interpretation of the experiment.

5. The problem of the spectrum of the hole system, in which the main level has several unoccupied states, can be raised in the strong-field approximation (the particles in the Landau level are conserved). All the results remain the same to within a shift of the origin of the energy measurement. Note that the holes are repelled from the positive impurity. If the main level has one vacancy, and the first Landau level has one electron or its spin is reversed, then new exciton- and spin-wave-type branches appear in the spectrum as a result of electron-hole interaction. The calculation of these branches has a lot in common with the problem of the Mott exciton in a strong field.[2,3] The important difference, however, is that the exchange contribution to the energy must be correctly calculated in our case. The spectrum appears to be continuous

because of the opposite sign of the charges of the interacting particle and hole. We shall give the results without going into details of the calculations:

$$E_{ex} = \frac{e\hbar H}{m^* c} + \frac{e^2}{2\epsilon l}\sqrt{\frac{\pi}{2}}\left\{1 - e^{-p^2/4}\left[\left(1 + \frac{p^2}{2}\right)I_0\left(\frac{p^2}{4}\right) - \frac{p^2}{2}I_1\left(\frac{p^2}{4}\right)\right] + \sqrt{\frac{2}{\pi}}\,pl^{-p^2/2}\right\}, \tag{7}$$

$$E_s = 2\mu^* H + \frac{e^2}{\epsilon l}\sqrt{\frac{\pi}{2}}\left[1 - e^{-p^2/4}I_0(p^2/4)\right] \tag{8}$$

Here I_0 and I_1 are the modified Bessel functions and the continuous quantum number p corresponds to the generalized momentum in l^{-1} units.[2] The excitonic branch (7) is analogous to the vibrational gap branch of the Wigner crystal.[1] For $p \ll 1$ $E_{ex} \approx e\hbar H/m^*c + e^2 p/2\epsilon l$, $E_s \approx 2\mu^* H + ap^2$. We also note that the upper limits of the spectra ($p \gg 1$) exceed $e\hbar H/m^*c$ and $2\mu^* H$, respectively.

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[1]H. Fukujama, Technical Report of ISSP, Ser. A, No. 993, 1979.
[2]L. G. Gor'kov and I. E. Dzyaloshinskiĭ, Zh. Eksp.Teor. Fiz. **53**, 717 (1967) [Sov. Phys. JETP **26**, 449 (1967)].
[3]I. V. Lerner and Yu. E. Lozovik, Zh. Eksp. Teor. Fiz. **78**, 1167 (1980) [Sov. Phys. JETP **51**, 588 (1980)].

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