Oscillations of a commensurable charge-density wave due to the influence of a constant electric field; an analogy with the Josephson effect

S. N. Artemenko and A. F. Volkov
Institute of Radio Engineering and Electronics, USSR Academy of Sciences

(Submitted 20 December 1980)

Pis'ma Zh. Eksp. Teor. Fiz. **33**, No. 3, 155–160 (5 February 1981)

It is shown theoretically that periodic current oscillations can be produced by a constant electric field in a conductor with a commensurable charge-density wave (CDW).

PACS numbers: 73.40.Gk

The conductivity of quasi-one-dimensional conductors, in which the superstructure—a charge-density wave (CDW)—is formed as a result of the Peierls transition has been intensively investigated in recent years. It has now been established theoretically and experimentally[1–3] that a CDW, which begins to move as a result of the action of a sufficiently large electric field E parallel to the conducting filaments, contributes to the conductivity. Since the CDW does not move in small fields, it can be recorded because of the interaction with the different defects or because of the commensurability effects (if the CDW constant is a multiple of the lattice constant). The incommensurable CDW, whose pinning is due to the interaction with the impurities, are formed in a quasi-one-dimensional metal $NbSe_3$, which has lately been investigated intensively. Several curious effects have been observed in the investigation of this material. It was shown in Refs. 4 and 5 noise generation begins to occur in the nonlinear region of the current-voltage (I-V) characteristic of $NbSe_3$, where the CDW begins to move, and Monceau *et al.*[6] discovered that a step appears in the I-V characteristic as a result of superposition of an *rf* field on the constant field E. These effects have been associated with the motion of a CDW due to the influence of the field E in the random potential of the impurities.

We shall analyze the pinning of a commensurable CDW and show that similar transient effects will also be observed when a CDW moves in a constant field E. However, coherent current (or voltage) oscillations, rather than noise oscillations, will be generated because the pinning in this case is attributable to the interaction of the CDW with the periodic crystal potential rather than with the random potential of the impurities.

In the calculation of the response of a conductor with a commensurable CDW, we shall generalize the kinetic equations for Green's functions, which were derived in Ref. 7 for the incommensurable case. It was shown in Ref. 1 that the system is described by the $n\times n$ matrix of the Green's functions G in the commensurable case of order n in which the electronic spectrum satisfies the condition $\epsilon_{p+nQ}=\epsilon_p$ (Q is the wave vector of the CDW), whereas the conductor with a CDW is described by the 2×2 matrix of the Green's functions in the incommensurable case. We shall examine

 0021-3640/81/03147-05$00.60

the case in which the energies $\epsilon_{p+m}Q$ for $m>1$, which were measured at the Fermi level, are such that $\epsilon_{p+m}Q$ are $\gg \Delta, T$ (Δ is the order parameter of the CDW and T is the temperature). Since the energies $\epsilon_{p+m}Q$ generally are of the order of the conduction-band width, this assumption can be easily satisfied if Δ and the temperature T_p of the Peierls transition are small compared with the band width.

Since the energies close to the Fermi surface are important in kinetics, we need only calculate the elements of the $G_{ik}(i,k \leqslant 2)$ matrix, which describe the states near the Fermi energy. We can derive an equation, which contains only these elements, from the equation for G. This equation has the form

$$\hat{G}^{-1}\hat{G} = 1,\ G^{-1} = \begin{pmatrix} \epsilon - \epsilon_p & \Delta + \gamma^*\Delta^* \\ \Delta^* + \gamma\Delta & \epsilon - \epsilon_{p+Q} \end{pmatrix}, \tag{1}$$

where $\gamma = \Delta^{n-2}/\epsilon_{p+2}Q\epsilon_{p+(n-1)}Q$. In the solution of the kinetic problems the potentials of the external fields must be included in the Hamiltonian in G^{-1}. We conclude from the relation (1) that in order to take the commensurability with $n>2$ into account, we must substitute $\gamma^*\Delta^*$ for Δ in the equations for Green's functions, which describe the incommensurable CDW.[7] The case $n=2$, in which $\Delta + \Delta^*$ is substituted for Δ because of commensurability, is a special case which will not be examined by us.

The commensurability changes the self-consistent condition[7] for Δ — instead of the Green's functions G_{12}, it contains the sum of the G_{n1} function and of all the $G_{m,m+1}$ functions, each of which, as can easily be shown, is equal to $\gamma^* G_{21}$. These additions must be taken into account in the equations in Ref. 7, which describe the linear response of the Peierls system with an incommensurable CDW. Moreover, since $|\gamma| \ll 1$, we can substitute the equilibrium Green's functions in the terms that take into account the commensurability effects. As a result, we can see that the expression for the current density is the same as that for an incommensurable CDW, and a new term, which is proportional to $|\gamma|$ and which describes the pinning, appears in the self-consistent equation, on which makes it possible to determine the connection between the velocity of a CDW $u = \chi/Q$ and the field E

$$\frac{1}{\omega_Q^2}\chi + n|\gamma|\sin(n\chi) = -\frac{i\lambda}{4\Delta}\int d\epsilon\, g_x, \tag{2}$$

where λ is the electron-phonon coupling constant, ω_Q is the frequency of a phonon with the momentum $Q, g_x = Sp(\hat{\sigma}_x)$, and $\hat{g}$ is the Green's function in which the phase factors were factored out. This function, which describes the linear response of an incommensurable CDW, was calculated in Ref. 7.

We shall consider the frequencies $\omega \ll \Delta, \tau_2^{-1}$, where τ_2 is the scattering time of the longitudinal momentum of an electron. The expression for the current at such frequencies has the form

$$j = \sigma_1 E + (\sigma_2 \hbar / e v \tau_2)\dot{\chi}, \tag{3}$$

where σ_1 and σ_2 describe the conductivities associated with the free electrons and with the motion of a CDW, respectively. The expressions for σ_i obtained in Ref. 7 have the following form in the limiting cases. a) A pure conductor at high temperatures $T \gg \Delta \gg \eta \gg \tau^{-1}$ (η is the characteristic width of the conduction band at right angles to the conducting filaments),

$$\sigma_1 = \sigma_N \left(1 - \frac{\Delta}{2T} A\right), \quad \sigma_2 = \sigma_N \frac{\pi}{4} \frac{\Delta}{T} \sqrt{\frac{\tau_2}{\tau}},$$

where $A \sim 1$, $1/\tau = 1/\tau_1 + 1/2\tau_2$, τ_1 is the time between the collisions during which the sign of the longitudinal momentum does not change and σ_N is the conductivity in the normal state. b) A pure conductor at low temperatures $\Delta \gg T \gg \eta \gg \tau^{-1}$; $\sigma_1 = \sigma_N 2\sqrt{\pi \tau_2 \tau T/V} \exp(-\Delta/T)$, $\sigma_2 = \sigma_N$. c) A gapless state (dirty material) $\eta \gg \tau^{-1} \gg \Delta$, $\sigma_1 = \sigma_N(1-a)$, $\sigma_2 = \sigma_N(a + b_0/2)$; here $b \ll a \ll 1$, where $a \sim \Delta^2 \tau_2/T$, $b \sim \Delta^2/T^2$ at $T \gg \eta$ and $a \sim \Delta^2 \tau_2/\eta$, $b \sim \Delta^2/\eta_2$ at $T \ll \eta$,

After substituting in Eq. (7) the g_x function from Ref. 7 the self-consistent equation has the form

$$\kappa^2 \ddot{\chi} + \frac{1}{\omega_0} \dot{\chi} + \sin(n\chi) = E/E_0, \tag{4}$$

where $\kappa = |\omega_Q \sqrt{n|\gamma|}|^{-1}$. In the same limiting cases,

$$\text{a)}\ E_0 = \frac{8\Delta T}{\pi e \hbar v} \sqrt{\frac{\tau}{\tau_2}} \frac{n|\gamma|}{\lambda}, \quad \omega_0 = \frac{4\Delta T \tau_2 n|\gamma|}{\hbar^2 \lambda \ln(\Delta/\eta)};$$

$$\text{b)}\ E_0 = \frac{2\Delta^2 n|\gamma|}{e\hbar v\lambda}, \quad \omega_0 = \frac{2\Delta T \tau_2 n|\gamma|}{\hbar^2 \lambda \ln T/\eta} l \frac{\Delta}{T};$$

$$\text{c)}\ E_0 = \frac{4\Delta^2 n|\gamma|}{e\hbar a v\lambda}, \quad \omega_0 = 4\Delta^2 \tau_2 n|\gamma| / (\hbar^2 a\lambda).$$

Equation (4) has the form of Josephson's equation for a point contact. This equation, together with the expression (3) for the current, determines the conductivity of the conductor with a commensurable CDW. It follows from these equations that many characteristic effects for Josephson junctions must be observed in this system. Specifically, it follows from Eq. (4) that if a constant field $E < E_0$ has an effect on the CDW, then the latter is stationary ($\chi = 0$) and the conductivity is equal to σ_1. The field effect in this case reduces to the loss of equilibrium of the CDW, $\chi = 0$. If the field is higher than the critical field E_0, then the CDW and hence the current will oscillate periodically in the sample. [1)] We have $\mathring{\chi} \approx \omega_0 E/E_0$ in the high-field limit and at $E \gg E_0$ and the dc conductivity is equal to $\sigma_1 + \sigma_2 \hbar\omega_0/eV\tau_2 E_0$, but the oscillation frequency of the phase and of the variable component of the current is equal to $n\omega_0 E/E_0$. If the field E contains a small variable $E_1 \sin \omega t$ in addition to the constant part, then the *I-V* characteristics will reveal the presence of a step with a slope equal to that of the I-V

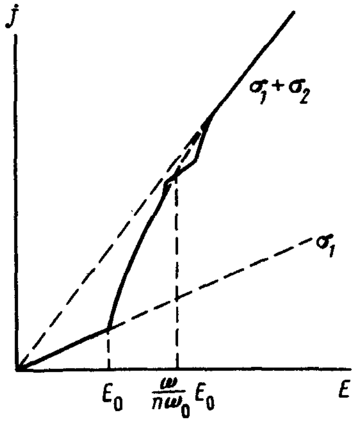

FIG. 1. *I-V* characteristic in the absence of irradiation (dashed curve) and in its presence at a frequency ω (solid curve).

characteristic at $E < E_0$. In contrast to the Josephson effect, the voltage at which the step appears is related to the frequency by the relation that contains not only e and $\hbar$ but also the parameters of the material. The center of the step for $\omega \gg n\omega_0$ is located at $E \approx \omega/n\omega_0 E_0$ and its amplitude is equal to $E_1 n\omega_0/\omega_1 \sqrt{1 + (\omega^2\omega_0^2)k^4}]$ (see Fig. 1).

The *I-V* characteristic in the nonlinear region depends on the conditions under which it was measured. For example, the *I-V* characteristic in this region has an *S*-shaped form for the specified current and $\kappa\omega_0 \ll 1$.

Equations (3) and (4) make it possible to determine the frequency dependence $\sigma(\omega)$ of the conductivity. It has the following form in a constant field $E < E_0$:

$$\sigma(\omega) = \sigma_1 + \sigma_2 \frac{\omega\hbar}{ev\tau_2 E_0} \frac{(\omega/\omega_0) + i(\kappa^2\omega^2 - n\cos n\chi_0)}{(\kappa^2\omega^2 - n\cos n\chi_0)^2 + \omega^2/\omega_0^2}, n\chi_0 = \arcsin\frac{E}{E_0}. \tag{5}$$

The characteristic values of the field E_0, of the frequency ω_0, and of the other quantities, which depend on different parameters of the material and on the temperature, can vary from one material to another within a very broad range, since the temperature of the Peierls transition, for example, in different materials varies in the range of several degrees to hundreds of degrees. The numerical estimates are rather difficult to perform for specific materials since many parameters are not known. An approximate estimate of the frequency ω_0 for a quasi-one-dimensional conductor TaS_3, in which a commensurable CDW with $n = 4$ is formed at 210 K,[8] shows that ω_0 may turn out to be a rather large value that lies in the microwave-to infrared-frequency range. We also note that a nonlinear *I-V* characteristic has been observed in TaS_3 in fields of about 100 V/cm.[8]

[1)]The amplitude oscillations of the CDW are small: $\delta\Delta \sim |\gamma|\Delta$.

[1]P. A. Lee, T. M. Rice, and P. W. Anderson, Solid State Comm. **14**, 703 (1974).
[2]N. P. Org and P. Monceau, Phys. Rev. **B 16**, 3443 (1977).
[3]L. P. Gor'kov and E. P. Dolgov, Zh. Eksp. Teor. Fiz. **77**, 396 (1979) [Sov. Phys. JETP **50**, 203 (1979)].
[4]R. M. Fleming and C. C. Grimes, Phys. Rev. Lett. **42**, 1423 (1979).
[5]M. Weger, G. Grüner, and W. G. Clark, Solid State Comm. **35**, 244 (1980).
[6]P. Monceau, J. Richard, and M. Renard, Phys. Rev. Lett. **45**, 43 (1980).
[7]S. N. Artemenko and A. F. Volkov, Zh. Eksp. Teor. Fiz. **81**, No 5 (1981).
[8]T. Sambongi, M. Ido, K. Tsutsumi, M. Yamamoto, T. Takoshima, and Y. Abe, Quasi-One-Dimensional Conductors, Proc. Int. Conf. Dubrovnik, 1978; Lecture Notes Phys. **95**, 349 (1979).

Translated by S. J. Amoretty
Edited by Robert T. Beyer