Infrared cutoff of transverse modes in the Yang-Mills theory at $T\neq 0$

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Nonvanishing screening of the effective interaction of transverse gluons in the Yang-Mills theory at $T\neq 0$ is obtained. The correlation between this fact and the phase transition in a quark-gluon plasma is discussed.

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An interest in the investigation of infrared properties of the Yang-Mills theory at $T\neq 0$ has increased considerably in recent years. This theory has a number of specific properties; in particular, its infrared divergences vary qualitatively at a finite temperature.[1,2] This is very important, since the screening of infrared divergences in the Yang-Mills theory can change dramatically our solution of the problem of "confinement" in quantum chromodynamics. Although the screening of infrared divergences for longitudinal modes of the effective interaction has been proved,[2] the question concerning the infrared behavior of the propagator of transverse gluons in the ($k_4=0$, $|\mathbf{k}|\to 0$) limit remains open. The authors of a number of papers[3] have expressed their views about the screening of transverse modes in this limit at momenta $|\mathbf{k}|\leqslant g^2T$; however, this has not been confirmed so far to a sufficient extent by appropriate calculations. We have attempted here to obtain a screening of transverse modes as a corollary of a certain self-consistent equation[4] which is derived in terms of the exact system of equations for Green's functions in QCD.[2,5] The screening of transverse modes, which determines the finite interaction radius of transverse gluons, was found to occur in the infrared region of small momenta.

The polarization operator in $SU(N)$ of the Yang-Mills theory at $T\neq 0$ is determined by two scalar functions[6]

$$\Pi_{ij}=\left(\delta_{ij}-\frac{k_i k_j}{\mathbf{k}^2}\right)A+\frac{k_i k_j}{\mathbf{k}^2}\,\frac{k_4^2}{\mathbf{k}^2}\,\Pi_{44}, \tag{1}$$

$$\Pi_{i4}=\Pi_{4i}=-\frac{k_i k_4}{\mathbf{k}^2}\,\Pi_{44},\quad (ij=1,2,3),$$

which satisfy the exact set of equations for the corresponding Green's functions.[2,5] A solution of this set of equations according to perturbation theory[2] leads specifically to screening of the longitudinal modes in the ($k_4=0$; $|\mathbf{k}|\to 0$) limit

$$\Pi_{44}(p_4=0;\quad |\mathbf{p}|\to 0)=\frac{1}{r_D^2}=\frac{g^2}{\beta^2}\,\frac{N}{3}\,. \tag{2}$$

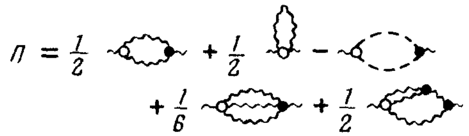
This has now been generally accepted. Here $\beta^{-1} = T$.

Asymptotic behavior of the A function in this limit within the framework of perturbation theory demonstrates that the effective interaction of transverse modes has a singular nature[7]

$$A(p_4 = 0;\ |\mathbf{p}| \to 0) = -\frac{7g^2N|\mathbf{p}|}{32\beta}, \tag{3}$$

which is quite remarkable, although it requires further study; for example, outside the scope of perturbation theory. For this purpose we shall use a self-consistent set of equations for the A and Π_{44} functions, which was obtained previously by Casado and Kalashnikov.[4]

The exact diagrammatic order for the polarization operator in Ref. 4


(4)

was calculated in a self-consistent manner by using the exact representation of the D function in terms of A and Π_{44} functions.[2] The vertex functions were also represented in a self-consistent way and the result of the calculations[4] was correlated with the Slavnov-Taylor identities.

The set of equations for A and Π_{44} functions is iterated independently in the framework of the simplest approximation. It is assumed that a nontrivial limit of the A function, which determines the screening of transverse modes, exists in the ($k_4 = 0$, $|\mathbf{k}| \to 0$) limit in addition to $\Pi_{44} \neq 0$. The exact equation for the A (k_4; $\mathbf{k}$) function,[4] with due regard for the fact that A ($k_4 = 0$; $|\mathbf{k}| \to 0) \neq 0$, approaches the limit and it is assumed that only one term with $\omega_n = 0$ must by retained in all the sums of the frequencies. A self-consistent equation for m^2 obtained in this manner

$$m^2 = \frac{2g^2m^2}{\beta} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\mathrm{p}^2(\mathrm{p}^2 + m^2)} \left[a + \frac{2p^2}{\mathrm{p}^2 + m^2}\right]$$

$$g^2 \frac{\Pi_{44}}{\beta} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\mathrm{p}^2(\mathrm{p}^2 + \Pi_{44})} \left[-1 + \frac{2\mathrm{p}^2}{\mathrm{p}^2 + \Pi_{44}}\right] \tag{5}$$

can be solved by using conventional methods. The gauge parameter in Eq. (5) is represented by α.

After subtracting the integrals, we can see that Eq. (5) reduces to a simpler form

$$m^2\left(1 - \frac{g^2N}{3\pi\beta}\,\frac{\alpha + 1}{2\sqrt{m^2}}\right) = 0 \tag{6}$$

A qualitative feature of Eq. (5) is that it has two solutions, which clearly resembles the systems that undergo a phase transition. A trivial solution of $m^2 = 0$ corresponds to a solution below the phase-transition point, where the infrared divergences are presumably effectively summed in such a way that the resulting behavior of the gluon propagator can be represented as $1/k^4$. This is the "confinement" phase. The solution of Eq. (6) is nontrivial above the phase-transition point[8]

$$m^2 = \frac{g^4(\alpha + 1)^2}{4\pi^2\beta^2}\left(\frac{N}{3}\right)^2, \tag{7}$$

This solution determines the screening of the effective interaction in the gluon-plasma phase. The phase transition has a disruptive nature, since the effective mass for $T = T_c$ vanishes abruptly. It is conceivable that the singularity of the effective potential determined in Ref. 7 can predict the phase transition in the gluon-plasma phase.

The interpretation of Eq. (6) given by us is by no means the only one. It may turn out that the phase with $m^2 = 0$ generally is not realized, and then an attempt to correlate the lack of screening of transverse modes with phase "confinement" can hardly be justified. It is important, however, that we have found for the first time a nontrivial second solution, although the value of m^2 in Eq. (7) changes because of allowance for higher approximations. The dependence of the obtained results on the gauge remains unexplained, although it does not encounter basic difficulties.

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