Self-modulation of radiation of a plasma cyclotron "maser"

P. A. Bespalov
Institute of Applied Physics, USSR Academy of Sciences

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It is demonstrated that a steady-state, single-mode lasing may be unstable in a many-level maser system.

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We show that a steady-state, single-mode lasing may be unstable in a many-level maser system (in contrast to a two-level system).

An example of a many-level system is a plasma cyclotron "maser," which can be used in a plasma trap with magnetic mirrors (for example, in the region of the earth's radiation belts). A relatively dense magnetized plasma and trap butts form a cavity for extraordinary electronic waves (whistlers) with frequencies $\omega \ll \omega_W$- for an electron cyclotron frequency.[1] The active material is a small addition of energetic electrons which have a slight effect on the dispersion equation but can enhance the whistlers. The distribution function of energetic electrons is anisotropic in a magnetic-mirror cyclotron; such a population inversion may cause a cyclotron instability (CI).[2]

If the pumping is relatively weak (i.e., if the source of energetic particles is sufficiently weak) under conditions of kinetic instability, then the self-consistent system of quasi-linear equations, which describes the nonlinear phase of the CI in a plasma maser, can be averaged over the period of oscillations of the particles between the mirror points T_b and of the waves between the cavity butts. The scattering of electrons at pitch angles by the whistlers, which in this case occurs with a negligible variation of the velocity modulus v, is described by the following equations when the frequency spectrum of the waves with $\mathbf{k}\|\mathbf{B}$ is relatively narrow[3]:

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial \kappa}\left(D\mathcal{E}\frac{\partial F}{\partial \kappa}\right) + J, \tag{1a}$$

$$\frac{\partial \mathcal{E}}{\partial t} = \left(\int_0^\infty \int_{\kappa_c}^{\kappa_m} K \frac{\partial F}{\partial \kappa}\, d\kappa\, dv\right)\mathcal{E} - \nu\mathcal{E}, \tag{1b}$$

where $\kappa = \sin\theta$, θ is the pitch angle of a particle for $W = W_{\min}$; the distribution function $F(t,\kappa,v)$, which was normalized to the total number of energetic electrons in the magnetic-field tube with a unit cross section at the level of the cavity butts $N = 2\pi\sigma\int_0^\infty\int_{\kappa_c}^1 T_b F v^3 \kappa\, d\kappa\, dv$, satisfies the boundary conditions at the loss cone $F|_\kappa = \kappa_c = 0$ ($\kappa_c = \sigma^{-1/2}$, where σ is the corkscrew ratio) and at the boundary of the interaction with the whistlers $\left.\frac{\partial F}{\partial \kappa}\right|_{\kappa=\kappa_m} = 0$. $J(\kappa,v)$ in Eq. (1a) is the strength of the

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source of particles in a magnetic tube, which is determined by the drift across the magnetic field, by the acceleration or by the injection of energetic electrons. The diffusion term in a many-level active material takes into account the transitions between the levels, which are associated with the scattering of particles at pitch angles. The larger is the average energy density of the waves $\mathscr{E}(t)$ in the magnetic-field tube, the faster is the diffusion of particles into the loss cone. $\mathscr{E}$, in turn, is determined from the averaged transport Eq. (1b), in which the average increment and the loss of whistler energy due to reflection from the butts and propagation, which are proportional to the absorption coefficient ν, are taken into account.

The stability of the steady-state of $F_0(\kappa,v)$ and $\mathscr{E}_0$ of the system (1), which corresponds to the constant intensity level and to the balance between the influx and emptying of particles into the loss is analyzed below; in general, the coefficients $D(\kappa,v)$, $K(\kappa,v)$, and $\kappa_m(v)$ are fairly complex functions.[3] For $\beta_* = (\omega_p v/\omega_B c)^2 \gg 1$, however, they depend weakly on κ.[1)] If we also assume that the corkscrew ratio is small and that the source delivers the particles with a small spread with respect to the v modulus, then D, K, and κ_m in Eq. (1) will be constant. Linearizing the system (1) for the process exp $\{\lambda,t\}$ near the steady state after converting to dimensionless variables, we obtain

$$\frac{\partial^2 f}{\partial \xi^2} + \mu^2 f = \epsilon j, \tag{2a}$$

$$\mu^2 \epsilon + f(\xi = 1) = 0. \tag{2b}$$

Here

$$\xi = (\kappa - \kappa_c)/(\kappa_m - \kappa_c), \quad \mu^2 = -\lambda(\kappa_m - \kappa_c)^2/D\mathscr{E}_0, \quad \epsilon = \mathscr{E}_\sim/\mathscr{E}_0,$$

$$f(\xi) = (\kappa_m - \kappa_c)^2 (K/D\mathscr{E}_0) \int_0^\infty F_\sim dv, \quad j(\xi) = (\kappa_m - \kappa_c)^4 (K/D^2\mathscr{E}_0^2) \int_0^\infty J\, dv.$$

Substituting in (2b) the solution of (2a), which satisfies the boundary conditions [see Eq. 1)] $f(0) = 0$, $\left.\frac{\partial f}{\partial \xi}\right|_{\xi=1} = 0$, we obtain a characteristic equation

$$\mu^3 \cos\mu = \int_0^1 j(\xi) \sin(\mu\xi)\, d\xi. \tag{3}$$

According to Eq. (2), the region $|\mathrm{Re}\mu| < |\mathrm{Im}\mu|$, in which $\mathrm{Re}\lambda > 0$, corresponds to the instability. If we consider a simple source $j = j_0 = \mathrm{const}$, then we get

$$\mu^4 \cos\mu = j_0 (1 - \cos\mu) \tag{4}$$

instead of Eq. (3). To determine the boundaries of the stability regions, we must substitute $\mu = a(1 \pm i)$ in Eq. (4) (a is a real number); it appears that the stability and the instability regions may be contiguous to each other, depending on the value of j_0, when

$$j_0 = \frac{4(\pi s)^4 \mathrm{ch}(\pi s)}{\mathrm{ch}(\pi s) - (-1)^s}, \qquad s = 1, 2, \ldots. \tag{5}$$

When $j_0 \gg 1$, Eq. (4) can be analyzed by using the perturbation method. Since $|\cos\mu| \gg 1$ in the region of possible instability, we find that

$$\lambda \sim -\mu^2 \approx \pm\, i j_0^{1/2} + j_0^{1/2} \exp\left\{-\left(\frac{j_0}{4}\right)^{1/4}\right\} \sin\left\{-\left(\frac{j_0}{4}\right)^{1/4}\right\}. \tag{6}$$

The stability and instability regions alternate periodically with increasing j_0, consistent with Eq. (5).

There is no instability when the angular dependence is arbitrary and j is sufficiently small, since it follows from Eq. (3) that $\lambda \sim -\mu^2 \approx -\int_0^1 j\xi d\xi$ for the solution that is closest to the stability boundary. The asymptotic expression for the solutions of (3) for relatively large j can be obtained by successive integration by parts and by using the perturbation method. The increment of modulation instability may be nonexponentially small for specific $j(\xi)$, and the stability and the instability zones do not alternate. For example, when $j(0) = 0$, $\left.\frac{\partial j}{\partial \xi}\right|_{\xi = 1} = 0$,

$$\lambda \sim -\mu^2 \approx \left.\left(\pm\, i j^{1/2} + \frac{1}{2j}\frac{\partial^2 j}{\partial \xi^2}\right)\right|_{\xi = 1}. \tag{7}$$

Specifically, $j \sim (4\xi - 8\xi^3 + 5\xi^4)$ corresponds to an instability.

If the source $J(\kappa, v)$ satisfies the same boundary conditions as F, we can solve the problem without assuming that D, K, and κ_m are constant by expanding F and J in eigenfunctions of the diffusion operator of Eq. (1a) with uniform boundary conditions. By using this method we can establish the fact that the frequency Ω_J and the increment γ_J of the modulation buildup of the waves for a sufficiently weak source are determined by the expressions

$$\Omega_J = \left(\int_0^{\infty}\int_{\kappa_c}^{\kappa_m} K \frac{\partial J}{\partial \kappa}\, d\kappa\, dv\right)^{1/2},$$

$$\gamma_J = \frac{1}{2\Omega_J^2}\int_0^{\infty}\int_{\kappa_c}^{\kappa_m} K \frac{\partial^2}{\partial \kappa^2}\left(D\mathscr{E}_0 \frac{\partial J}{\partial \kappa}\right) d\kappa\, dv,$$

consistent with Eq. (7). We must take into account when using Eqs. (6) and (7) that $j(\xi)$, which repeats the angular dependence of the source J, is inversely proportional to it since $\mathscr{E}_0 \sim J$.

Thus, the steady-state mode of the cyclotron excitation of waves in a single-mode plasma maser with many levels due to the inhomogeneity of the medium is unstable relative to the perturbations with the frequency $\Omega_J \sim J^{1/2}$ at a certain (sufficiently small) power and angular dependence of the source. The excitation of the intensity-level peaks and of the particle flux was typical of the laboratory devices and of the geometric trap.[4] Our analysis illustrates one of the natural causes of the onset of such a process.

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[1)] At $\bar{\beta}_* \gg 1$ the frequency shift, which has the maximum average increment, can be ignored.

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