

Self-induced transparency of surface polaritons

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A self-induced transparency (SIT) of surface polaritons under resonance conditions with oscillations in the transition layer is predicted. The shape of the surface soliton (surface “ 2π ” pulse) is determined and the dependence of its duration on the dielectric constants of the contacting media and on the characteristics of the layer is investigated. The layer is described in the context of the model of a two-dimensional gas of the two-level systems.

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The self-induced transparency (SIT) in gaseous and condensed media, which was predicted in Ref. 1, has been thoroughly investigated (see, for example, Ref. 2 and the literature cited therein). However, the SIT in surface electromagnetic waves heretofore has not been discussed. Because of the increased research in surface spectroscopy, this possibility has now become worth exploring.

We report in this paper the first results of the formulation of the SIT theory for surface polaritons. Suppose that there is a thin transition layer in an isotropic polarization $\mathbf{P}(x,t)\delta(z)$ in the plane $z=0$ at the interface ($z=0$) of the isotropic media $\mathbf{I}(z>0, \epsilon = \epsilon_1)$ and $\mathbf{II}(z<0, \epsilon = \epsilon_2)$. If the surface current is taken into account, then the boundary conditions for $z=0$ for the field components in the H wave propagating

along the x axis [$\mathbf{H} = (0, H, 0)$, $\mathbf{E} = (E_x, 0, E_z)$] must have the form

$$H^{(1)} - H^{(2)} = -\frac{4\pi}{c} \frac{\partial P_x}{\partial t}, \quad E_x^{(1)} - E_x^{(2)} = 0. \quad (1)$$

After conversion to Fourier components in x and t , the wave equation of H , i.e., the equation

$$\left(-\epsilon_i \frac{\omega^2}{c^2} + k^2\right) H(k, \omega, z) - \frac{d^2 H(k, \omega, z)}{dz^2} = 0,$$

where $\epsilon_i = \epsilon_1$ for $z > 0$ and $\epsilon_i = \epsilon_2$ for $z < 0$, has the following solution which decreases as $z \rightarrow \pm \infty$:

$$H(k, \omega, z) = \begin{cases} H^{(1)}(k, \omega) e^{-\kappa_1 z}, & z > 0, \quad \kappa_1 = \left(k^2 - \epsilon_1 \frac{\omega^2}{c^2}\right)^{1/2} \\ H^{(2)}(k, \omega) e^{\kappa_2 z}, & z < 0, \quad \kappa_2 = \left(k^2 - \epsilon_2 \frac{\omega^2}{c^2}\right)^{1/2} \end{cases} \quad (2)$$

It follows from the Maxwell equation $\text{rot } \mathbf{H} = \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t}$ as $z \rightarrow \pm 0$ that

$$\begin{aligned} -i \frac{\omega}{c} \epsilon_1 E_x^{(1)}(k, \omega) &= \kappa_1 H^{(1)}(k, \omega) \\ -i \frac{\omega}{c} \epsilon_2 E_x^{(2)}(k, \omega) &= -\kappa_2 H^{(2)}(k, \omega) \end{aligned} \quad (2a)$$

so that if we take Eq. (1) into account and set $E_x^{(1)}(k, \omega) = E_x^{(2)}(k, \omega) = E_x(k, \omega)$

$$\left(\frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2}\right) E_x(k, \omega) = -4\pi P_x(k, \omega) \quad (3a)$$

or in the coordinate representation

$$\int_{-\infty}^{\infty} \frac{d\omega dk}{(2\pi)^2} f(k, \omega) E_x(k, \omega) e^{i(kx - \omega t)} = -4\pi P_x(x, t), \quad (3b)$$

where

$$f(k, \omega) \equiv \left(\frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2}\right).$$

The relations (3a) and (3b) obtained on the basis of the Maxwell equations establish a correlation between the boundary value of the electric field intensity and the transition-layer polarization and are valid for any linear or nonlinear material correlation between them.

We are interested in the solutions of Eq. (3b) in the pulse form

$$E_x(x, t) = e(x, t) \exp[i (Qx - \Omega t)], \quad (4)$$

which satisfies the "slowness" condition $\left| \frac{\partial e}{\partial x} \right| \ll Qe, \left| \frac{\partial e}{\partial t} \right| \ll \Omega e$. If the $f(k, \omega)$ function varies slowly in the spectral pulse width, we can use below the expansion

$$f(k, \omega) = f(Q, \Omega) + \frac{\partial f}{\partial \omega} \bigg|_{\substack{\omega = \Omega \\ k = Q}} (\omega - \Omega) + \frac{\partial f}{\partial k} \bigg|_{\substack{\omega = \Omega \\ k = Q}} (k - Q) \quad (5)$$

in the calculation of the left-hand side of Eq. (3b). We shall assume, moreover, that the pulse duration is short compared with the relaxation time of two-level molecules in the transition layer and, for simplicity, we shall also assume that the frequency Ω exactly matches the transition frequency in the two-level systems. As we know,^{1,2} in this case

$$P_x(x, t) = ip(x, t) \exp[i (Qx - \Omega t)] \quad (6)$$

$$p(x, t) = \frac{1}{2} d n_0 \sin \Psi (x, t),$$

where d is the dipole moment of the transition, n_0 is the density of two-level molecules in the transition layer, and $\Psi \equiv d \int_{-\infty}^t e(x, t') dt'$. Substituting Eqs. (5) and (6) in Eq. (3b) and separating the real and the imaginary parts, we obtain two equations

$$f(Q, \Omega) = 0, \quad \frac{\partial e}{\partial t} + v_{\text{lim}} \frac{\partial e}{\partial x} = -4\pi \left(\frac{\partial f}{\partial \omega} \right)_{\substack{\omega = \Omega \\ k = Q}}^{-1} p(x, t), \quad (7)$$

where $v_{\text{lim}} = d\Omega/dQ$. The first equation in (7), which determines the dependence $\Omega = \Omega(Q)$ for the carrier frequency in Eq. (4), coincides with that [see Eq. (3b)] for the surface polariton, if the transition layer is ignored. The second equation for the steady-state pulses, i.e., those with $e(x, t) = e(\tau)$, where $\tau = t - x/u$ and u is the pulse rate, has the form

$$\frac{d^2 \Psi}{d\tau^2} = \frac{1}{\tau_p^2} \sin \Psi, \quad (8)$$

where

$$\frac{1}{\tau_p^2} = \frac{2\pi d^2 n_0}{\frac{v_{\text{lim}}}{u} - 1} \left(\frac{\partial f}{\partial \omega} \right)_{\substack{\omega = \Omega \\ k = Q}}^{-1} \quad (9)$$

The derivative $\left(\frac{\partial f}{\partial \omega} \right)_{\substack{\omega = \Omega \\ k = Q}} > 0$ in the region of the surface polariton (see, for example, Ref. 3) and the inequality $u < v_{\text{lim}}$ follows from Eq. (9). The solution of Eq. (8), which

corresponds to the solution (see Ref. 1), is $e(\tau) = \frac{2}{d\tau_p} \cosh^{-1}(\tau/\tau_p)$, so that the field in the plane $z = 0$ [see Eq. (4)] turns out to be completely defined. The relations (2) and (2a) as well as the relation for the electric-field intensity analogous to (2), together with the Maxwell equations, completely define all the field components in the pulse for arbitrary x , t , and z .

We emphasize that the obtained surface SIT pulse, in contrast with the pulse analyzed in Ref. 1, has a certain small radiation width which depends on the capacity of the body waves to radiate into the half-space $z > 0$ (for specificity, we assume that $\epsilon_1 > 0$ and $\epsilon_2 < 0$) by the tail of the spectral decomposition of the pulse for such k and ω whose $\kappa_1^2(k, \omega) < 0$.

The method discussed makes it possible to analyze the SIT in thin, macroscopic films and in waveguides.

¹S. L. McCall and E. L. Hahn, Phys. Rev. **183**, 457 (1969).

²L. Allen and J. Eberl, *Opticheskiĭ rezonans i dvukhurovnevye atomy* (Optical Resonances and Two-Level Atoms), Mir, Moscow, 1978.

³V. M. Agranovich and A. G. Mal'shukov, Optics Comm. **11**, 169 (1974).

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