Adiabatic inversion of population and nonlinear absorption of light by accelerated atoms

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New nonlinear effects in the absorption of high-power radiation by accelerated atoms or ions, which give rise to a critical dependence of the absorption and population on the parameter $\delta = |d_{12}E_0|^2/\hbar^2 ka$ (where d_{12} is the dipole moment of the transition, k is the wave vector, and a is the acceleration) are analyzed. These effects are important in the kinetics of ionic lasers.

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Nonlinear effects in the spectra of accelerated atoms have been analyzed in Refs. 1–3 in the context of perturbation theory in a field E_0 with due account taken of the relaxation γ of the levels. We shall analyze this effect in a field E_0 of arbitrary magnitude, but ignore the relaxation, i.e., we shall analyze it at a sufficiently small value of the parameter γ.[1)]

We shall use standard equations for the density matrix of a two-level system[3] and assume that this is a steady-state, spatially homogeneous problem. We can easily see that these equations are equivalent to the following system of equations for the amplitudes b_1 and b_2 of the system:

$$\left.\begin{aligned} ia\frac{\partial b_1}{\partial v} &= (kv - \Delta\omega)\, b_1 + V b_2 \\ ia\frac{\partial b_2}{\partial v} &= V b_1 \end{aligned}\right\} \qquad (1)$$

where v is the velocity of an atom, $\Delta\omega = \omega - \omega_{12}$ is the frequency difference ω of laser radiation relative to the frequency ω_{12} of the atomic transition, and $V \equiv d_{12}E_0/\hbar$.

Equations (1) are solved for the initial condition $b_2(v = v_0) = 0$ and the averaging is performed over the Maxwell's distribution $f(v_0)$ of the initial velocities v_0.

We can see from Eq. (1) that this problem is completely analogous to the theory of nonadiabatic Landau-Zener transitions (Ch. 19 in Ref. 4), i.e., the derivatives with respect to v play the role of the time derivatives and the Doppler frequency shifts kv play the role of the linearly varying terms.

The solutions of the $b_2(v_0, \infty)$ system, which determine the probability of the transition $W_{12}(v_0) = |b_2(v_0, \infty)|^2$, can be expressed, as we know,[4] in terms of the parabolic-cylinder functions. They show that $W_{12}(v_0)$, plotted as a function of v_0, has a sharp discontinuity at the point $v_0 = v_\omega \equiv \Delta\omega/k$ (at the point of "intersection of the terms"), whose effective width is $\Delta v \approx \max\{V/k, \sqrt{a/k'}\}$.

We write the expression for absorptive power (see Ref. 3)

$$P(\omega) = Q\hbar\omega \int_{-\infty}^{\infty} dv_0\, W_{12}(v_0) f(v_0), \qquad (2)$$

where Q is the pumping to the level 1.

Assuming that the thermal velocity v_T of atoms satisfies the condition $v_T \gg \Delta v$ and that the discontinuity of the $W_{12}(v_0)$ function is sharp, we obtain

$$P(\omega) = Q\hbar\omega\, \Phi\left(\frac{\Delta\omega}{k}\right) |\, b_2(-\infty, +\infty)\,|^2, \qquad (3)$$

where

$$\Phi(v) = \int_{-\infty}^{v} f(v)\, dv.$$

Using the well-known expression for the probability of nonadiabatic transitions in the Landau-Zener model: $|b_2(-\infty, +\infty)|^2 = 1 - c^{-2\pi\delta}$, we finally obtain

$$P(\omega) = Q\hbar\omega\, \Phi\left(\frac{\Delta\omega}{k}\right)[1 - \exp(-2\pi V^2/ka)]. \qquad (4)$$

The physical meaning of Eq. (4) is self-evident: only those particles, whose velocity is lower than the resonance velocity $v_\omega \equiv \Delta\omega/k$ during excitation, participate in the absorption; the $\Phi(\Delta\omega/k)$ factor determines the statistical weight of these particles and the $1-e^{-2\pi\delta}$ factor determines the absorption probability at the "point of intersection" v_ω.

The Φ function in Eq. (4) is independent of the relaxation γ if the relaxation is unessential when the system moves from the point v_0 to the point v_ω, which corresponds to the condition $\frac{a}{\gamma v_T} \gg 1$. Otherwise, the $\Phi(\Delta\omega/k,\gamma)$ distribution in Eq. (4) is more complicated. Note that the localization of the transition at the resonance point $kv = \Delta\omega$, which corresponds to the structure of Eq. (4), must have a large "modulation index[5]" that reduces to the condition $kv_T^2/a \gg 1$ in the case under consideration, which can easily be satisfied in practice.

For $V^2/ka \ll 1$ Eq. (4) coincides with that in Sec. 21 of Ref. 3, if the relaxation in the latter equation goes to zero.

For $V^2/ka \gtrsim 1$ the absorption is strongly nonlinear.

Another curious conclusion can be drawn from the behavior of the average probability W_{12} at the level 2

$$< W_{12}(v_0, v) > = \int_{-\infty}^{\infty} dv_0 f(v_0)\gamma \int_{v_0}^{\infty} \frac{dv}{a} |b_2(v_0, v)|^2 \approx \phi\left(\frac{\Delta\omega}{kv_T}\right)\left(1-e^{-2\pi\delta}\right). \tag{5}$$

We can see that this probability tends to unity when $\Delta\omega \gtrsim kv_T$ and $2\pi V^2/ka \gg 1$,[2)] i.e., the atom, which was originally in the state 1 turns out to be in the totally unoccupied state 2. The state 1 in this case is depopulated. We call this effect adiabatic population inversion. In fact, according to the picture of the terms in the Landau-Zener model for a large V, the system moves along the "adiabatic" terms, which corresponds to the transitions with the probability 1 between the original (adiabatic) terms corresponding to the levels 1 and 2 of the original atom.

These effects are of vital interest in the kinetics of ionic lasers. In fact, they have been observed at $V^2/ka \gg 1$ and at $\gamma V/ka \lesssim 1$, consistent with our discussion. If $V \sim \sqrt{ka}$ in the nonlinear region, we arrive at the condition $\gamma^2/ka \lesssim 1$. This parameter, for example, for an argon laser, is equal to 1/3 according to the estimates in Ref. 1.

In conclusion, we note that the effects examined above must have a critical (exponential) dependence on the ion mass in order to accelerate the ions in an external electric field.

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[1)]A more thorough analysis shows that allowance for γ only complicates the spectral dependence and does not change the qualitative conclusions. The case discussed below corresponds to the criterion $\gamma|d_{12}E_0|/\hbar ka \ll 1$.

[2)]The divergence of the integral with respect to v, which follows from the behavior of the $|b_2(v_0,v)|^2$ function as $v \to \infty$, is unimportant, since the integral is cut off at $v_{max} \sim a/\gamma$, if γ is taken into account. It is important that the condition $v_{max} \gg \Delta v$, which reduces to the condition $\gamma V/ka \ll 1$ indicated above for

$V^2/ka \gg 1$, should be satisfied.

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