Dislocations of the wavefront of a speckle-inhomogeneous field (theory and experiment)

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(Submitted 25 December 1980)

Pis'ma Zh. Eksp. Teor. Fiz. **33**, No. 4, 206–210 (20 February 1981)

The average number N of wavefront dislocations per cm^2 of beam cross section is calculated for a speckle-inhomogeneous monochromatic field: $N \sim (\Delta\theta)^2/\lambda^2$, where $\Delta\theta$ is the angular divergence. The complex amplitude of the field goes to zero at these points. The wavefront dislocations of the speckle-inhomogeneous field are recorded experimentally by observing the interference with an auxiliary reference beam; their average number is in good agreement with the theoretical prediction.

PACS numbers: 42.10.Dy, 42.10.Jd

The structure of the wavefront of optical radiation is of great interest because of the capability of beam transformation and correction by means of adaptive flexible mirrors.[1] The problem of wavefront dislocations is attracting special attention.

The appearance of wavefront dislocations is unavoidable in a monochromatic polarized beam with adequately developed inhomogeneities of complex amplitude. For a real monochromatic field $E_{\text{real}}(\mathbf{r}, z, t)$ with a central propagation direction along the z axis, we introduce the slow complex amplitude $\mathbf{E}(\mathbf{r}, z)$ by the definition

$$E_{\text{real}}(\mathbf{r}, z, t) = \frac{1}{2}\{\mathbf{e}\, e^{-i\omega t + ikz} E(\mathbf{r}, z) + \mathbf{e}^* e^{i\omega t - ikz} E^*(\mathbf{r}, z)\},$$

where $k = 2\pi/\lambda$ and λ is the wavelength in the medium. The wavefront surface is defined by the equation

$$kz + \arcsin(\text{Im}E(\mathbf{r}, z)/|E(\mathbf{r}, z)|) = \text{const}. \qquad (1)$$

We can see from this that the systematic features of the surface (1) coincide with the zeros of the modulus $|E(\mathbf{r},z)| = (\sqrt{(ReE)^2 + (ImE)^2})$. At a given beam cross section $z =$

const the zeros of the amplitude $|E|$ appear if the equations

$$\mathrm{Re}\, E(x, y, z) = 0, \qquad \mathrm{Im} E(x, y, z) = 0, \tag{2}$$

are satisfied simultaneously; in general, these equations are satisfied at discrete points on the (x, y) plane. For beam propagation along z these points describe certain curved trajectories. If a given line $|E(\mathbf{r},z)| = 0$ corresponds to simple zeros, i.e., not multiple zeros in Eqs. (2), then the phase (1) of the complex amplitude $E(\mathbf{r},z)$ can be shifted by 2π or -2π as a result of by-passing this line along a closed contour. This corresponds to the surface of a wavefront with a right-handed or left-handed screw dislocation, respectively. It is easy to understand that for field propagation along z the screw dislocations can be generated and annihilated only as pairs with opposite spirality signs. A somewhat different treatment of the dislocations of the wavefront is given in Refs. 2–4.

For a random complex field we can obtain an explicit expression for the average number of zeros NS in the area S,

$$NS = < \int \delta (E_1)\, \delta (E_2) dE_1 dE_2 > , \tag{3}$$

where $E_1 = Re\, E(x,y)$ and $E_2 = Im\, E(x,y)$. The Jacobian $G \equiv |E_{1x}E_{2y} - E_{1y}E_{2x}|$, where $E_{1x} \equiv \partial E_1/\partial x$, etc., appears when we integrate with respect to the variables $dxdy$ instead of $dE_1 dE_2$. For the averaging it is necessary to specify the six-dimensional probability density $W(E_1, E_2, E_{1x}, E_{1y}, E_{2x}, E_{2y})$. For a speckle-inhomogeneous field with Gaussian statistics this gives

$$N = \frac{2\pi}{\lambda^2} \sqrt{\det \hat{C}}\,; \qquad C_{ik} = \overline{\theta_i \theta_k} - \bar{\theta}_i \bar{\theta}_k . \tag{4}$$

Here the bar above the symbols denotes averaging over the normalized angular spectrum $j(\theta_x, \theta_y)$, which characterizes the "gray" divergence of the speckle-inhomogeneous field:

$$\overline{\theta_i^n} = \int \theta_i^n\, j(\theta_x, \theta_y)\, d^2\theta .$$

Here "gray" denotes the irregular part of the divergence, i.e., the part that cannot be eliminated by inserting continuous lenses into the beam.

We recorded in this experiment the wavefront dislocations of a speckle-inhomogeneous laser beam and measured their density as a function of the "gray" divergence. The interference of the speckle-inhomogeneous wave $E(\mathbf{r},z)$ with a plane reference wave E_0, which is tilted by an angle α much greater than the divergence of the investigated wave, was used to visualize the dislocations. It is easy to prove that the dislocations of the wavefront $E(\mathbf{r},z)$ correspond to production (or annhiliation) of an interference fringe. The interference fringes are continuous when two fields without dislocations interfere. The experimental setup is shown in Fig. 1. The one-mode, linearly polarized beam of the LG-38 He-Ne laser was divided into two beams—a reference E_0 beam and a signal $E(\mathbf{r},z)$ beam. The signal beam was expanded to a diameter of $\approx$4.7 mm by the telescope and transmitted through the diaphragmmed phase plate P (which is standard for wavefront-inversion experiments[5]). As a result, the signal beam acquired severe phase distortions, but the amplitude within the aperture limits of the

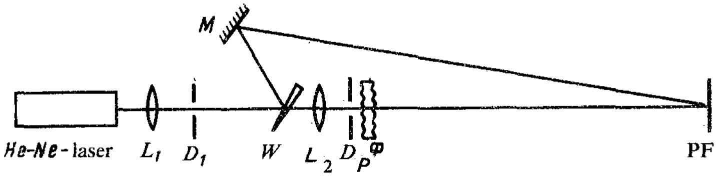

FIG. 1. Schematic of the experiment. The reference wave was produced by reflection from the wedge W and from the blind mirror M. D_1 is the spatial filter at the focus of the first lens of the telescope L_1L_2.

diaphragm was almost constant. The phase modulation was transformed to amplitude modulation during its propagation–the signal beam acquired a pronounced speckle structure. The phase plate was sufficiently strong in order to eliminate the isolated regular component in the distorted wave; the radiation divergence was

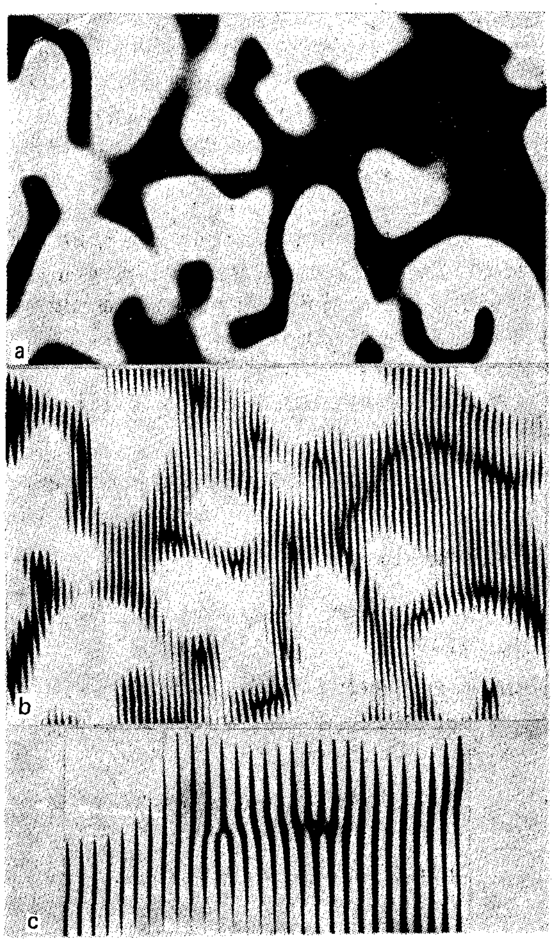

FIG. 2. Photographs of a portion of the speckle structure of the field without the reference wave (a) and in the presence of it (b); (c) magnified image of two dislocations with opposite spirality sign. One of them corresponds to an extra interference fringe (when the fringes are traced from the bottom to the top) and the other to the disappearance of a fringe.

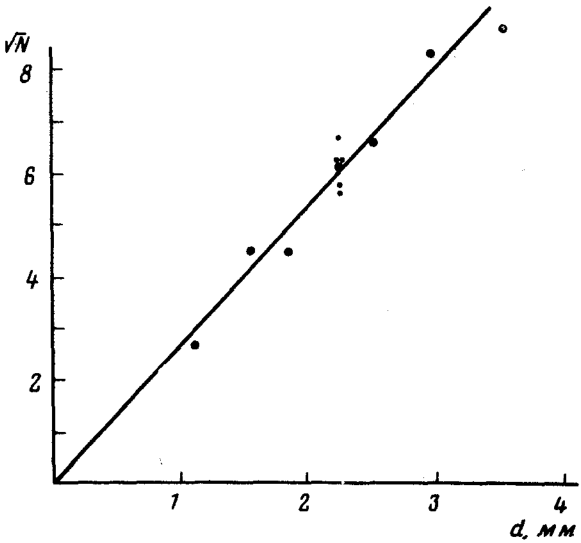

FIG. 3. Experimental points and theoretical dependence of $(\sqrt{N})$ on the diameter d, which is proportional to the gray divergence of the signal wave.

$\Delta\theta(FWe^{-1}M) = 6.7\times10^{-3}$. The reference and signal beams interfered at the photographic film PF, which was located at a distance $R = 9.5$ m from the phase plate.

Figure 2 shows the following photographs: a) a part of the speckle pattern of the signal wave $E(\mathbf{r},z)$, b) the same part as in a) but modulated by interference with the E_0 wave. The points of interference-fringe production and annihilation correspond to wavefront dislocations and, therefore, to zeros of the amplitude $E(\mathbf{r},z)$. By comparing photographs a) and b), we can prove that a zero of the speckle-inhomogeneous field amplitude does not occur in all the minima of the speckle pattern. Figure 2c shows a large-scale image with two dislocations of opposite sign.

To measure the density of dislocations N (cm^{-2}) of the speckle field, we counted the number of points of interference-fringe production and annihilation in a 2.3×3.3 – cm^2 film frame. The gray divergence of the field $E(\mathbf{r},z)$ in the frame plane, which was determined by the viewing angle of the diaphragm D_2, $\theta_0 = d/2R$, where d is the diameter of the diaphragm, was varied by varying the diaphragm diameter within the limits $5.8\times10^{-5}\leqslant\theta_0\leqslant1.8\times10^{-4}$ rad. In this case the angular spectrum on the film had the shape of a "table": $j(\theta) = 1/\pi\theta_0^2$ for $|\theta|\leqslant\theta_0$ and $j(\theta) = 0$ for $|\theta| > \theta_0$. For such an angular spectrum we obtain $N = \pi\theta_0^2/2\lambda^2$ from Eq. (4). Figure 3 shows the experimental points of the dependence of $(\sqrt{N})$ on the diameter d. The spread of the $(\sqrt{N})$ values for different speckle-patterns is shown for a diaphragm diameter $d = 2.2$ mm. The solid line corresponds to the theoretical dependence (4).

Thus, we have recorded the wavefront dislocations of a speckle-inhomogeneous laser beam and calculated and measured their density as a function of the "gray" divergence. The experimental results are in agreement with theory within an accuracy of 5%. The wavefront dislocations reduce the effectiveness of its correction or inversion by means of continuous felxible mirrors. In fact, it is impossible to transform the mirror surface into a surface with a screw dislocation without destroying the continuity conditions. In contrast, the wavefront surfaces of the acoustic wave, which also have this type of dislocations, are reflecting surfaces in the case of wavefront inversion by stimulated scattering.[5]

The authors thank A. L. Gyulamiryan and I. I. Sobel'man for valuable discussions.

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Translated by Eugene R. Heath
Edited by S. J. Amoretty