

# Gravitational distortions in neutron-optical systems and their reduction by means of nonuniform magnetic field

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The problem of gravitational distortions in neutron-optical systems for ultracold neutrons (UCN) is analyzed. The use of a nonuniform magnetic field with a vertical magnetic-field gradient is proposed for the reduction of such distortions. The compensating action of the magnetic field from a current loop is examined as a model.

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Very low energy neutrons-ultracold neutrons (UCN) and very cold neutrons (VCN)-have become available for experiments during the last decade. According to the terminology developed currently, the UCN are neutrons that are totally reflected from the boundary of the material at all angles of incidence. The faster neutrons, for which the refractive index differs greatly from unity, are call VCN. Both of the indicated properties of very slow neutrons make it possible, in principle, to produce neutron-optical devices based both on reflection and on refraction. It is hoped that the development of the new field neutron optics will lead to the construction of a neutron microscope.<sup>1-3</sup>

We know that the refractive index of a neutron wave in matter has the form

$$n^2 = 1 - \lambda^2 \frac{Nb}{\pi} \quad (1)$$

where  $N$  is the number of nuclei per unit volume of matter,  $\lambda$  is the wavelength, and  $b$  is the coherent scattering length.

In general, when a certain region of space has the potential  $U(\mathbf{r})$ , the refractive index can be written as follows:

$$n^2 = 1 - \lambda^2 \frac{2m}{h^2} U(\mathbf{r}) \quad (2)$$

where  $m$  is the neutron mass. The  $U(\mathbf{r})$  potential of a neutron can be comprised of three parts, corresponding to the number of interactions in which the neutron participates

$$U(\mathbf{r}) = U_{\text{nuc}}(\mathbf{r}) + \vec{\mu} \mathbf{B}(\mathbf{r}) + mgz \quad (3)$$

where  $\mu$  is the magnetic moment of a neutron,  $B$  is the magnetic induction,  $g$  is the acceleration of gravity,  $U_{\text{nuc}} = 2\pi h^2 Nb / m$  is the potential of nuclear interaction of a neutron with the medium, and the other two terms describe the interaction of a neutron with the magnetic field and with the earth's gravitational field. The effect of gravity must be taken into account because of the small neutron energy (a UCN with a

characteristic energy of  $10^{-7}$  eV can be raised approximately 1 meter in the earth's gravitational field).

Since the force of gravity bends the neutron trajectories in an optical system, the neutrons of different energies are focused at different points and the system produces chromatic aberrations.<sup>1</sup> This effect can be formally described by assigning a refractive index to the vacuum in a gravitational field.<sup>4,5</sup> After substituting the value  $U = mgz$  in Eq. (2), we obtain

$$n^2 = 1 - \lambda^2 \frac{2m^2gz}{h^2} = 1 - \frac{2gz}{v_0^2}, \quad (4)$$

where  $v_0$  is the neutron velocity at a certain height  $z = 0$ .

To compensate for the gravitational chromatism resulting from the focusing of neutrons, Steyerl and Schütz<sup>3,6</sup> proposed the use of a zone mirror combination of a concave mirror with a zone diffraction grating. The gravitational chromatism in this case is compensated for by the intrinsic chromatism of the diffraction system. The compensation is obtained in such a device when the source is situated at a specified distance from the image in the neighborhood of a certain wavelength  $\lambda$ .

There is evidently another solution of this problem, which ensures a compensation for all the wavelengths  $\lambda$ . This can be achieved if the refractive index is independent of the coordinates in the entire region of space under investigation, which is occupied by the optical system. Therefore, the following equation is valid in the absence of matter:

$$\vec{\nabla} U = \vec{\nabla} (\vec{\mu} \mathbf{B}) + mg\mathbf{k} = 0, \quad (5)$$

where  $\mathbf{k}$  is the unit vector of the  $z$  axis.

If the neutrons are transmitted through the region with a magnetic field under adiabatic conditions, which can easily be achieved for UCN and VCN, then the projection of the magnetic moment of a neutron on the induction vector will be conserved and Eq. (5) can be rewritten as follows:

$$\mu \vec{\nabla} |\mathbf{B}| + mg\mathbf{k} = 0 \quad (6)$$

or in the cylindrical coordinate system

$$\mu \frac{\partial |\mathbf{B}|}{\partial z} = mg, \quad (7)$$

$$\frac{\partial |\mathbf{B}|}{\partial \rho} = 0 \quad (8)$$

After substituting the numerical values in (7), we obtain  $\partial |\mathbf{B}|/\partial z = 170 \text{ G/cm}$  for the essential gradient.

Of course, the conditions (7) and (8) cannot be satisfied exactly in the entire region. However, these equations, in principle, are compatible at least at some point, since they contain the value  $B$ , rather than the induction-vector component. The prob-

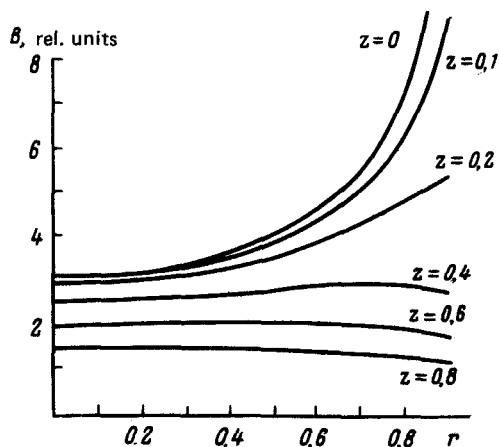


FIG. 1. The field of a unit-radius current loop as a function of the radius for different heights  $z$  above the current plane. The dependence of the field on the radius is weak for  $z = 0.6$ .

lem, therefore, reduces to the question whether a magnetic system, which satisfies the conditions (7) and (8) at some point for moderately strong dependence of the value  $\partial|\mathbf{B}|/\partial z$  on the coordinates and which also satisfies the condition

$$\frac{\partial|\mathbf{B}|}{\partial\rho} \ll \frac{\partial|\mathbf{B}|}{\partial z} \quad (9)$$

can be determined.

The simplest approach here is to analyze the field produced by the current loop. Figures 1, 2, and 3 show the values of  $B$ ,  $\partial|\mathbf{B}|/\partial z$ , and  $\partial|\mathbf{B}|/\partial\rho$  of the unit-radius loop along which the unit current flows at different values of  $z$  and  $\rho$ . We can see that  $\partial|\mathbf{B}|/\partial\rho$  curves intersect the  $x$  axis in the region  $z \approx 0.6$  and at different values of  $\rho$ , and the values of the radial gradient are small for small values of  $\rho$ . Thus, there are points at which (8) is satisfied and regions in which (9) is valid. The question of constant  $\partial|\mathbf{B}|/\partial z$  in space is generally linked with the absolute dimensions of the system. The roughly estimate the compensation efficiency of gravitational distortions, we performed a numerical calculation of the image size of the point source of neutrons in the plane in which the light (or neutrons in the absence of gravitation) must be focused

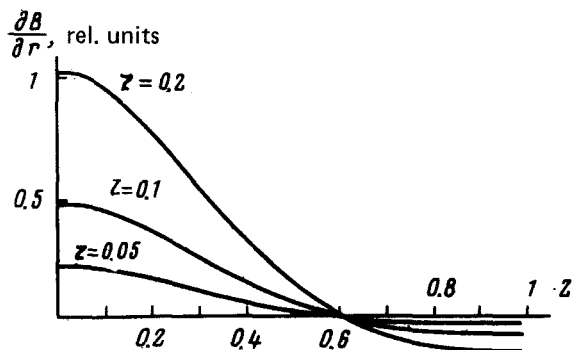


FIG. 2. Radial field gradient of the current ring, as a function of  $z$  for different values of  $r$ . There is no radial gradient at  $z \approx 0.6$ .

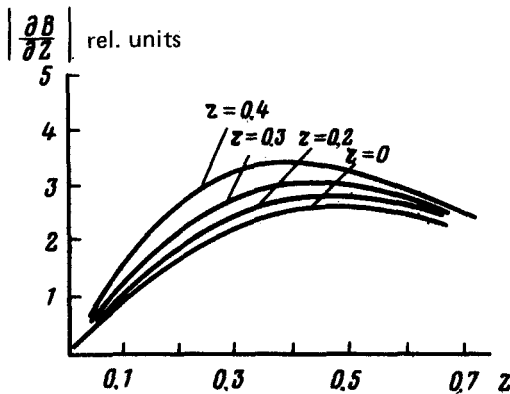


FIG. 3. Vertical field gradient of the current ring as a function of  $z$  for different  $r$ . The gradient varies moderately in the region  $0.5 \leq z \leq 0.7$  and  $r \leq 0.2$ .

after reflection in a spherical mirror.

The magnetic and optical systems had the following parameters: the radius of the current loop  $R = 20$  cm, the radius of the spherical mirror  $r = 5$  cm, the spacing between the source and the mirror was equal to 7 cm, and the aperture  $2\alpha = 1^\circ$ . The optical system was placed near the point  $z = 0.6$ ,  $\rho = 12$  cm. The spectrum of the neutron velocities was uniform  $3 \leq v \leq 5$  m/sec. The calculation showed that the image size for an optical system of this size can be reduced by a factor of approximately 50 by "turning on" the magnetic field of required magnitude. The residual smearing is associated primarily with insufficient uniformity of the  $\partial |\mathbf{B}|/\partial z$  component and the effect of the nontrivial component  $\partial |\mathbf{B}|/\partial r$  is small. This can be improved by increasing the diameter of the current loop.

The necessary current is equal to  $\sim 10^5$  A for the radius of the loop indicated above. This current can easily be achieved in a superconducting system if the cross section of the winding is not too large ( $7\text{--}10$  cm<sup>2</sup>). To obtain the required induction gradient, the current must be increased proportionally to  $R^2$  if the dimensions of the loop are increased.

It can evidently be assumed, therefore, that the gravitational distortions can be reduced by using a nonuniform magnetic field. The technological difficulties associated with the construction of the magnetic system are closely linked with the desirable degree of compensation. The current level of technology, however, is capable of reducing the gravitational distortions significantly.

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