Quantum oscillation effects in cylindrical conductors in a weak magnetic field

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Quantum oscillation effects in cylindrical conductors of radius R in a longitudinal magnetic field **H** are analyzed. Quantum oscillations, which are analogous to the Kosevich and Lifshitz oscillations for electrons in a plate, were obtained in fields for which the Larmor orbit radius $r_H > R$.

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Quantum magneto-dimensional oscillations (MDO) of thermodynamic and kinetic quantities, which are significantly different from the oscillations of the electron characteristics of massive samples, occur in magnetic fields H when the diameter of the electron orbits is equal to or greater than the conductor size. The MDO in metal films, which were predicted by Kosevich and Lifshitz,[1] were observed in a study of the electrical conductivity of antimony whiskers.[2] The MDO have a different nature in cylindrical conductors, as observed by Brandt, Gitsu, Nikolaeva, and Ponomarev[3] in the measurement of the magnetoresistance of cylindrical bismuth single crystals in fields

$$H < H_0 \equiv cP_F/eR, \tag{1}$$

where R is the radius of the cylinder, e is the modulus of the electron charge, c is the velocity of light, and P_F is the Fermi momentum.

The samples examined in Ref. 3 have a high degree of specularity for surface scattering of electrons.

The MDO in cylindrical conductors in a longitudinal magnetic field for conditions of specular reflection of electrons by the metal boundry were investigated by Dingle,[4] who obtained many frequencies that were equidistant with respect to H in the fields $H \ll H_0$. We show in this paper that only one frequency of the many Dingle frequencies remains at intermediate temperatures T (or because of electron scattering by impurities). The quantum oscillations are also equidistant in H in the fields $|H - H_0| \ll H_0$. In intermediate fields (1) the period of the oscillations ΔH depends on H and R, analogously to the Kosevich and Lifshitz oscillations[1] for electrons in a plate. The relative change of ΔH with the field for adjacent periods is of the order of $\hbar/P_F R \ll 1$. An analysis of the quantum oscillations has been carried out for conduction electrons with a square isotropic dispersion law. In metals with complex Fermi surfaces the oscillation picture is complicated. Several frequencies, which are attributable to electrons of different levels of the Fermi surface, can be produced.

The oscillations examined in this paper are caused by electrons from the outer orbits. Relative to these electrons, the contribution to the oscillations of the electrons that slide along the cylinder surface[5] is of the order of

$$(\hbar/P_F R)^{3/4} \exp[-\pi (T + T_D) mR/\hbar P_F] << 1,$$

where m is the effective electron mass and T_D is the Dingle temperature. The electrons at the nonextremal cross sections of the Fermi surface and the quantum oscillations corresponding to them[6] are important only for diffuse scattering of electrons at the sample boundary. This effect does not occur in the case of specular reflection. An analogous (negative) result was previously[7] obtained for plates.

In the temperature region,

$$\Delta\epsilon_F < T << \epsilon_F \ , \tag{2}$$

where $\Delta\epsilon_F$ is the distance between the quantum energy levels in the vicinity of the Fermi level ϵ_F, the complicated oscillation dependence of the thermodynamic and kinetic quantities on H in the interval of fields (1) is greatly simplified by determining the fundamental harmonic with the largest amplitude. Figure 1 shows the electron orbits in the coordinate (a) and momentum (b) space, which cause the given oscillations. The period of the oscillations ΔH is determined by the truncated extremal area of the Fermi-surface cross section (see Fig. 1b):

$$\Delta H = \frac{\hbar c}{eR^2} \Phi\left(\frac{eHR}{c\, P_F}\right), \tag{3}$$

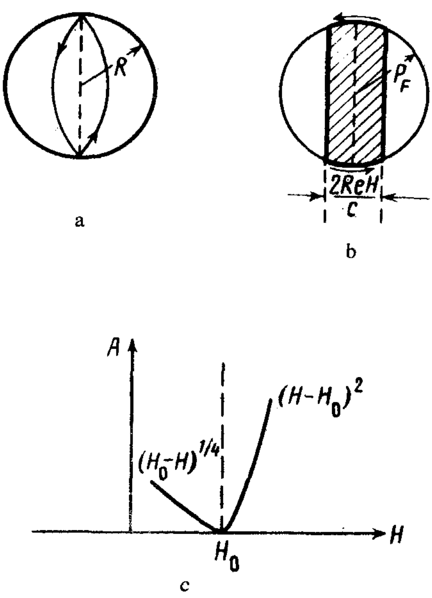

FIG. 1. Electron orbits in the coordinate space (a) and in the momentum space (b), which determine the quantum magneto-dimensional oscillations of the electron characteristics of cylindrical conductors in a longitudinal field. The oscillation amplitude A has a characteristic feature (c) in the vicinity of the cutoff field H_0.

where

$$\Phi(x) = x^2\left[\frac{1}{2} - \frac{1}{\pi}\arcsin\sqrt{1-x^2} - \frac{x}{\pi}\sqrt{1-x^2}\right]^{-1}. \tag{3'}$$

As H increases the period of the oscillations ΔH decreases monotonically and at $H = H_0$ it corresponds to a quantum of magnetic flux through the cylinder cross section.

In the temperature interval (2) the quantum oscillations of the electrical conductivity (see Ref. 8) are proportional to the differential magnetic susceptibility χ_{osc}. The corresponding expression for the fundamental harmonic $\chi_{osc}^{(1)}$ with the period (3) was obtained for an arbitrary magnetic field. In weak fields $H \ll H_0$ it has the following form:

$$\chi_{osc}^{(1)} = \frac{4}{9\pi}\left(\frac{eR}{c}\right)^2 \frac{P_F T}{\hbar^3}\left(\frac{H}{H_0}\right)^2 \cos\left[4\frac{P_F R}{\hbar}\left(1 - \frac{1}{6}\left(\frac{H}{H_0}\right)^2\right)\right]\exp\left(-4\pi\frac{mRT}{\hbar P_F}\right). \tag{4}$$

As $H \to 0$, the amplitude of the fundamental harmonic $\chi_{osc}^{(1)}$ goes to zero, so that in the expression for χ_{osc}, which is determined by the sum of the harmonics, it is necessary to take into account the next largest harmonic $\chi_{osc}^{(2)}$

$$\chi_{osc}^{(2)} = \frac{3}{4\pi}\left(\frac{eR}{c}\right)^2 \frac{P_F T}{\hbar^3}\sin\left(3\sqrt{3}\frac{P_F R}{\hbar}\right)$$

$$\times \cos\left(\frac{3\sqrt{3}}{4}\frac{eHR^2}{\hbar c}\right)\exp\left(-3\sqrt{3}\pi\frac{mRT}{\hbar P_F}\right). \tag{5}$$

The period of the $\chi_{osc}^{(2)}$ oscillations, which is constant in a forward field $\Delta H = 8\pi\hbar c/3\sqrt{3}eR^2$, coincides with one of the oscillation periods obtained by Dingle.[4]

In the vicinity of the field $H_0(H_0 - H \ll H_0)$ the amplitude of the $\chi_{osc}^{(1)}$ oscillations is proportional to $(H_0 - H)^{1/4}$, and the period of the oscillations is constant in a forward field: $\Delta H = 2\hbar c/eR^2$. The number n of such oscillations satisfies the condition $2n\hbar/P_F R \ll 1$. In strong fields $H \gg H_0$ the χ_{osc} oscillations coincide with the oscillations of the magnetic susceptibility of massive samples, and they decrease as $(H - H_0)^2$ as $H \to H_0$ (see Fig. 1c). The aforementioned asymptotes, which are proportional to $(H_0 - H)^{1/4}$ and $(H - H_0)^2$, respectively, are combined in close proximity of the point H_0 that satisfies the condition $|1 - \frac{H}{H_0}| \lesssim (\hbar/P_F R)^{1/2}$.

The isolation of one harmonic of the frequencies can occur both with an increase in the temperature [see Eq. (2)] and because of electron scattering by impurities. The influence of impurities can be taken into account by replacing T by $T + T_D$ with the appropriate Dingle temperature T_D in the given equations. Under the conditions (2) the magnitude of the χ_{osc} oscillations of the magnetic susceptibility relative to its smooth part χ is of the order of

$$\left|\frac{\chi_{osc}}{\tilde{\chi}}\right| \sim \frac{P_F R}{\hbar} \frac{mR(T+T_D)}{\hbar P_F} \exp\left(-4\pi \frac{mR(T+T_D)}{\hbar P_F}\right). \qquad (6)$$

The sensitivity of the amplitude[3] of the two-frequency oscillations, which are equidistant in H, to a change in temperature indicates that the condition (2) (for $T \to T + T_D$) is satisfied. The number of frequencies is determined by the number of electron ellipsoids in bismuth, which are nonequivalent for the given magnetic-field direction. The specular reflection of conduction electrons at the sample boundary determines the frequencies which are equidistant in H in the fields $H \ll H_0$ and $|H - H_0| \ll H_0$. The oscillation period ΔH depends on H in the intermediate fields. The relative variation of ΔH is very small ($\sim \hbar/P_F R$) in the adjacent periods; therefore, a special investigation may have to be conducted to determine this dependence.

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