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Deconfinement transition for nonzero baryon density in the field correlator method

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Deconfinement phase transition due to disappearance of confining colorelectric field correlators is described using nonperturbative equation of state. The resulting transition temperature $T_c(\mu)$ at any chemical potential μ is expressed in terms of the change of gluonic condensate ΔG_2 and absolute value of Polyakov loop $L_{\rm fund}(T_c)$, known from lattice and analytic data, and is in good agreement with lattice data for $\Delta G_2 \approx 0.0035\,{\rm GeV}^4$. E.g. $T_c(0) = 0.27; 0.19; 0.17\,{\rm GeV}$ for $n_f = 0, 2, 3$ respectively.

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1. Phase transition at nonzero μ and dynamics of quark gluon plasma (QGP) is now of great interest because of impressive results of heavy ion experiments, see [1] for a recent review and references. The topic calls for a nonperturbative (NP) treatment of QCD degrees of freedom at nonzero T and μ, which is especially important for not very large T, μ. Below we are using the NP approach based on field correlator method (FCM) [2], which was applied to nonzero T in [3, 4].

The main advantage of FCM is a natural explanation and treatment of dynamics of confinement, as well as the deconfinement transition [3, 4], in terms of Color Electric (CE) $D^E(x), D_1^E(x)$ and Color Magnetic (CM) Gaussian (quadratic in $F^a_{\mu\nu}$) correlators $D^H(x), D_1^H(x)$.

The correlators $D^E(x)$ and $D^H(x)$ ensure confinement in the planes $(4,i)$ and (i,k) respectively, $i,k = 1,2,3$ so that standard string tension $\sigma^E \equiv \sigma = \frac{1}{2}\int D^E(x)d^2x$, and spatial string tension $\sigma^H \equiv \sigma_s = \frac{1}{2}\int D^H(x)d^2x$. Correlators D_1^E, D_1^H contain perturbative series, and D_1^E plays an important role in that it contributes to the modulus of the Polyakov line; at $T \ge T_c$ one has [5]

$$L^{(V)}_{\rm fund} = \left|\frac{1}{N_c}\left\langle {\rm tr}\, P \exp ig\int_0^\beta A_4 dz_4\right\rangle\right| = \exp\left(-\frac{V_1(\infty,T)}{2T}\right), \quad (1)$$

with the static $Q\bar{Q}$ potential

$$V_1(r,T) = \int_0^\beta d\nu(1-\nu T)\int_0^r \xi d\xi D_1^E(\sqrt{\xi^2+\nu^2}). \quad (2)$$

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Deconfinement phase transition in this language is the disappearance of $D^E(x)$ (and σ^E) at $T \ge T_c$, while $L^{(V)}_{\rm fund}$ and $L^{(V)}_{\rm adj} = (L^{(V)}_{\rm fund})^{9/4}$ are nonzero there[2)].

However the disappearance of $D^E(x) = \frac{1}{N_c}\langle {\rm tr}\, E_i(x)\Phi(x,y)E_i(y)\rangle$ implies vanishing of a part of vacuum energy density,

$$\varepsilon_{\rm vac} = \frac{\beta(\alpha_s)}{16\alpha_s}\langle (F^a_{\mu\nu})^2\rangle = -\frac{(11-\frac{2}{3}n_f)}{32}G_2,$$
$$D^E(0) + D_1^E(0) + D^H(0) + D_1^H(0) = \frac{\pi^2}{9}G_2 \quad (3)$$

and G_2 is the gluon condensate [6]. At $T = 0$, $D^E = D^H, D_1^E = D_1^H$ and for $T > 0$ both D^H, D_1^H do not change till $T \approx 2T_c$, while D^E disappears at $T \ge T_c$ [7] in agreement with the deconfinement mechanism suggested in [4].

Particle data [7] and analytic study [8] imply that $D_1^{(E,H)}(x) \approx 0.2 D^{(E,H)}(x)$, therefore one expects that $\Delta G_2 = G_2(T < T_c) - G_2(T > T_c) \approx \frac{1}{2}G_2(T < T_c) \approx \frac{1}{2}G_2^{st}$, where $G_2^{st} \approx 0.012\ {\rm GeV}^4$ [6] (see [9] for a recent gauge-string duality treatment of G_2^{st} yielding $G_2^{st} \approx (0.01 \pm 0.002)\ {\rm GeV}^4$).

This ΔG_2 taken as the change of free energy (pressure) across the phase boundary will be our basic element in finding the phase transition curve $T_c(\mu)$ below.

[2)]The subscript V in $L^{(V)}_{\rm fund}$ is to distinguish from $L^{(F)}_{\rm fund}$ calculated on the lattice with singlet free energy $F^1_{Q\bar{Q}}(\infty,T)$ replacing $V_1(\infty,T)$ in (1). It is clear that $L^{(F)}$ contains all bound states in addition to the ground state (selfenergy) $V_1(\infty,T)$, hence $F^1_{Q\bar{Q}} \le V_1$. We shall ignore the difference in the first approximation in what follows and write $L_{\rm fund}$.

To this end we introduce in the next section the NP equation of state of QGP derived recently in [10], and express $T_c(\mu)$ in terms of ΔG_2 and $L_{\rm fund}(T_c)$.

Taking for the latter the lattice or analytic value, one obtains a set of curves $T_c(\mu)$ for $n_f = 0, 2, 3$ depending on the only parameter ΔG_2. These resulting curves and their end points $T_c(0), \mu_c(0)$ are discussed in conclusion.

2. In the NP approach to the QGP in [10] one introduces in the first approximation the interaction of single quarks and gluons with the vacuum, which is called the Single Line Approximation (SLA), leaving pair and triple, etc... correlations to the next steps. As a result one obtains in SLA the pressure $P_q^{\rm SLA}$ of quarks (and antiquarks) and $P_g^{\rm SLA}$ of gluons which are expressed through $L_{\rm fund}$, namely [10]:

$$p_q \equiv \frac{P_q^{\rm SLA}}{T^4} = \frac{4N_c n_f}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^4}L^n_{\rm fund}\varphi_q^{(n)}\cosh\frac{\mu n}{T}, \tag{4}$$

$$p_{gl} \equiv \frac{P_{gl}^{\rm SLA}}{T^4} = \frac{2(N_c^2-1)}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^4}L^n_{adj}, \tag{5}$$

with

$$\varphi_q^{(n)}(T) = \frac{n^2 m_q^2}{2T^2}K_2\left(\frac{m_q n}{T}\right) \approx 1 - \frac{1}{4}\left(\frac{n m_q}{T}\right)^2 + ... \tag{6}$$

In (4), (5) it was assumed that $T \lesssim 1/\lambda \cong 1\,\text{GeV}$, where λ is the vacuum correlation length, e.g. $D_1^{(E)}(x) \sim \sim e^{-|x|/\lambda}$, hence powers of L_i^n, see [10] for details.

With few percent accuracy one can replace the sum in (5) by the first term, $n = 1$, and this form will be used below for p_{gl}, while for p_q this replacement is not valid for large μ/T, and one can use instead the form equivalent to (4),

$$p_q = \frac{n_f}{\pi^2}\left[\Phi_\nu\left(\frac{\mu - V_1/2}{T}\right) + \Phi_\nu\left(-\frac{\mu + V_1/2}{T}\right)\right], \tag{7}$$

where $\nu = m_q/T$ and

$$\Phi_\nu(a) = \int_0^\infty \frac{z^4 dz}{\sqrt{z^2+\nu^2}}\frac{1}{e^{\sqrt{z^2+\nu^2}-a}+1}. \tag{8}$$

Eqs. (7), (5) define p_q, p_{gl} for all T, μ and m_q, which is the current (pole) quark mass at the scale of the order of T.

Using (4)–(8) we can define the pressure $P_{\rm I}$ in the confined phase, and $P_{\rm II}$ in the deconfined phase, taking into account that vacuum energy density in two phases $\varepsilon_{\rm vac}$ and $\varepsilon_{\rm vac}^{\rm dec}$ respectively contributes to the free energy, and hence $|\varepsilon_{\rm vac}|, |\varepsilon_{\rm vac}^{\rm dec}|$ to the pressure. Denoting the hadron gas pressure in the confined phase as $P_{\rm hadron}$ one has

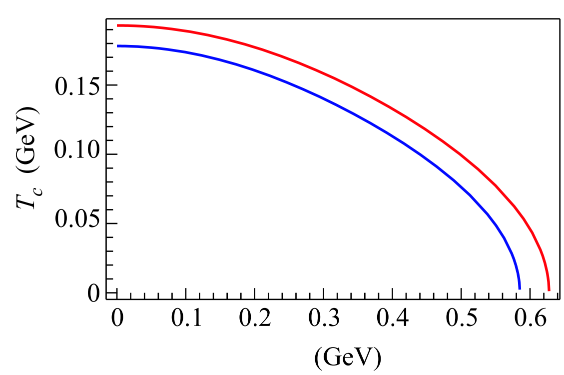
$$P_{\rm I} = |\varepsilon_{\rm vac}| + P_{\rm hadron}, \quad P_{\rm II} = |\varepsilon_{\rm vac}^{\rm dec}| + (p_{gl} + p_q)T^4. \tag{9}$$

From $P_{\rm I}(T_c) = P_{\rm II}(T_c)$ one obtains $T_c(\mu)$, neglecting $P_{\rm hadron}$ in the first approximation

$$T_c = \left(\frac{(11 - \frac{2}{3}n_f)\Delta G_2}{32(p_{gl}(T_c) + p_q(T_c))}\right)^{1/4}. \tag{10}$$

In (10) enter only two parameters; ΔG_2 and $L_{\rm fund}(T_c) = \exp(-\kappa/T_c)$, $\kappa \equiv \frac{1}{2}V_1(\infty, T_c) \cong \frac{1}{2} \times \times F^1_{Q\bar{Q}}(\infty, T_c)$.

The latter can be found in 3 different ways: 1) from the direct lattice measurements [11] of $P^1_{Q\bar{Q}} \approx 0.5\,\text{GeV}$; 2) from analytic calculation of D_1^E in [8], which yields $V_1(\infty, T < T_c) \approx 6\alpha_s(M_0)\sigma/M_0 \approx 0.5\,\text{GeV}$ with $M_0 \cong \cong (0.8-1)\,\text{GeV}$ lowest gluelump mass [12]; 3) from lattice calculations of D_1^E at $T > T_c$, [7, 13], which according to (2) yields $V_1(\infty, T_c) \approx 0.4 \div 0.6\,\text{GeV}$. Therefore one can fix $V_1(\infty, T_c) = 0.50(5)\,\text{GeV}$ ($\kappa = 0.25\,\text{GeV}$) and this value is independent of n_f [11]. As a result $T_c(\mu)$ is a function of only ΔG_2 and for each value of ΔG_2 one finds a set of curves for $n_f = 2, 3, ...$ We choose $\Delta G_2 \approx \frac{1}{2}G_2^{st}$ and in Figure the curves computed numerically from (10) for $n_f = 2, 3$ are shown for $\Delta G_2 \approx 0.00341\,\text{GeV}^4$ and zero quark pole masses.



The phase transition curve $T_c(\mu)$ from Eq.(10) (in GeV) as function of quark chemical potential μ (in GeV) for $n_f = 2$ (upper curve) and $n_f = 3$ (lower curve) and $\Delta G_2 = 0.0034\,\text{GeV}^4$

The end points $T_c(0)$ and $\mu_c(0)$ can be found analytically e.g. for $T_c(\mu)$ with $\sim 5\%$ accuracy one has (expanding (10) in p_{gl}/p_q)

$$T_c(\mu) = T_c(0)\left(1 - C\frac{9\mu^2}{T_c^2(0)}\right), \quad C = 0.0110(3), \tag{11}$$

with

$$T_c(0) \approx \frac{1}{2}T^{(0)}\left(1+\sqrt{1+\frac{\kappa}{T^{(0)}}}\right),$$
$$T^{(0)} = \left(\frac{(11-\frac{2}{3}n_f)\pi^2\Delta G_2}{384 n_f}\right)^{1/4}. \quad (12)$$

For $\Delta G_2 = 0.00341\,\mathrm{GeV}^4$ one obtains $T_c(0) = (0.27; 0.19; 0.17)$ GeV for $n_f = 0, 2, 3$ respectively, which agrees well with numerous lattice data, see [14] for reviews. The value of $C = 0.011$ is inside the scattered set of lattice values [14].

Another end point, $\mu_c(0)$ can be found from the asymptotics of (8), $\Phi_0(a \to \infty) = \frac{a^4}{4} + \frac{\pi^2}{2}a^2 + ...$, which yields (for $m_q = 0$) $\mu_c(0) = \kappa + (48)^{1/4}T^{(0)}$ and for the same ΔG_2 as above one gets $\mu_c(0) = (0.63; 0.58)$ GeV for $n_f = 2, 3$ One can check, that the derivative in T, $d\mu_c(T)/dT$ vanishes at $T = 0$.

3. The phase curve $T_c(\mu)$ in Figure is in reasonable agreement with lattice data at least for $\mu \lesssim 0.25$ GeV, see [15] for review and references. Two important points are to be discussed here: 1) order of transition and possible critical point 2) approximations and assumptions of the present work.

1). The vacuum transition of our approach is evidently of the first order at least in the leading (SLA) approximation used for (10), and does not contain any critical points. This is in agreement with lattice $n_f = 0$ data, but the lattice results for $n_f = 2, 3$ depend on masses, discretization and are not fully conclusive. The softening of transition for $n_f > 0$ in our approach is explained by the increasing role of $P_{\mathrm{hadron}}(T)$ for $n_f > 0$ near T_c, which suppresses the specific heat and makes the curve $P(T)$ more smooth. In addition there is interparticle (e.g. qq and qqq) interaction disregarded above in the first approximation which can soften the transition.

The chiral transition in our approach is caused by the deconfinement, since both $\langle\bar{q}q\rangle$ and f_π are expressed via $D^E(x)$ [16] and vanish together with it, in agreement with lattice data, see e.g. [14]. The Polyakov loop is a good (approximate) order parameter for $n_f = 0 (n_f > 0)$ since at $T < T_c$ it is expressed via $D^E(x)$ and vanishes (strongly decreases) (see Eq. (6) of [5]).

2). In our derivation of (10)–(12) it was assumed: a) that the only important part of QGP dynamics is the interaction with the NP vacuum –SLA; b) it is assumed that vacuum fields do not depend on T, μ in the phase diagram, except at the phase boundary where the shift ΔG_2 occurs; in particular neither ΔG_2 nor $L_{\mathrm{fund}}(T_c)$ depend on μ. The latter point is partly supported by lattice data [17]. In general this picture of rigid vacuum is based on the notion of the dilaton scale m_d of vacuum fields, which can be associated with the 0^{++} glueball mass around 1.5 GeV and therefore for all external parameters (like μ or T) much less than m_d vacuum fields are fixed.

Another argument in favor of rigid vacuum is that all dependence on μ and n_f does not appear in the lowest order of $1/N_c$ expansion, since it comes from the quark loops. (Note that nevertheless $T_c(0)$ differs strongly for $n_f = 0$ and $n_f = 2, 3$; even through ΔG_2 was kept fixed, and this successful prediction of $T_c(0) = 0.27$ GeV and 0.19 GeV respectively can be considered as another support of our picture). Several things were not taken into account. Quark masses are included trivially via Eqs.(6), (7) and this can be checked *vs* lattice data. Phase transition near $\mu_c(0)$ can be complicated due to strong qq and qqq interaction, which is not taken into account above and will be discussed elsewhere (see also [10]); therefore the possibility of color superconductivity is not commented here.

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