

Quantum teleportation of a unknown N-qubit W-like state

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We propose a nontrivial protocol to teleport a unknown N-qubit W-like state. The consumed resource is only $(N-1)$ shared ebits and $2(N-1)$ bits of classical communication, while the technique involves only controlled-NOT gates and single-qubit measurements/operations. The rule for reconstruction of the desired state at the receiving station is worked out explicitly in the most general case of an arbitrary $N \geq 3$. The protocol is within the reach of present technologies.

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Historically, the notion of entanglement was introduced by Schrödinger in 1935 [1], long before the dawn of the relatively young field of quantum information. Nowadays, entanglement has been served as a useful (in some cases unreplaceable) resource in quantum information processing and quantum computing. So, as a necessity, understanding and employing entangled state become more and more important. Besides well-understood bipartite entangled states, there also exist multipartite entangled ones that, though less-understood, play a very significant role in quantum networking. Two inequivalent representatives of multipartite entangled states are the GHZ [2] and the W [3] states which cannot be converted to each other by local unitary operations and classical communication. Compared with the GHZ states, less work has been done for the W ones. Schemes for generation of W states are proposed in [4] and applications of them are suggested in [5]. Especially, N-qubit W states (for $N > 10$) have been shown to exhibit more robust violation of local realism, than the GHZ ones [6]. The W states, by definition [3], are maximally entangled states. In the case of nonmaximal entanglement we refer to them as W-like states which are also important in processing quantum information. For example, remote symmetric entangling [7, 8] and perfect teleportation of a qubit [9] strictly require W-like but not W states.

In this work we deal with teleportation of a unknown N-qubit W-like state of the form

$$|\mathrm{W}_N\rangle_{12...N} = = (x_1|10...0\rangle + x_2|01...0\rangle + \cdots + x_N|00...1\rangle)_{12...N} \quad (1)$$

using shared ebits in terms of EPR pairs as the quantum channels. To our best knowledge, such a kind of task has not been touched upon so far. As is well-known, an arbitrarily general N-qubit state can always be teleported by the universal protocol [10] using N ebits, $2N$ bits and N Bell measurements (BMs). However, so much resource may be luxury for a particular state that does not span the entire 2^N-dimensional Hilbert space. For example, a unknown N-qubit GHZ-like state $|\mathrm{GHZ}_N\rangle_{12...N} = (\alpha|00...0\rangle + \beta|11...1\rangle)_{12...N}$ can be teleported just via 1 ebit and 2 bits, independent of N [11]. Because the W-like state (1) lives in a subspace spanned by $|10...0\rangle_{12...N}$, $|01...0\rangle_{12...N}$, ..., and $|00...1\rangle_{12...N}$ (i.e., the subspace dimension is $N < 2^N\ \forall N \geq 3$), one expects a cheaper cost to teleport it. Indeed, we shall show that the required numbers of shared ebits and communicated bits are only $(N-1)$ and $2(N-1)$, respectively.

The main technical challenge of quantum teleportation is commonly associated with BMs [12], whose outcome is a two-qubit Bell state. To avoid BM several modified teleportation schemes have been proposed [13]. However, the schemes in [13] concern only the continuous-variable system or high-Q cavity system. So far, we have not seen teleportation scheme without BM in the linear optics system. In [14] a way was found to implement efficient quantum computation using only linear optics, photo-detectors and single-photon sources. Subsequently, realization of photonic controlled-NOT (CNOT) gate was reported experimentally [15]. Motivated by that, we propose here a scheme to teleport the state (1) using only CNOTs and simple single-qubit measurements, i.e., BMs are not necessary.

Suppose first that Alice is asked to teleport to her remote Bob a 3-qubit W-like state

$$|\mathrm{W}_3\rangle_{123} = (x_1|100\rangle + x_2|010\rangle + x_3|001\rangle)_{123}, \qquad (2)$$

where $|x_1|^2+|x_2|^2+|x_3|^2=1$ with no information on an individual x_n. At this aim, Alice and Bob need a priori share 2 ebits in terms of 2 identical EPR pairs of the form

$$|\mathcal{B}\rangle_{A_iB_i} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_iB_i},\ i=1,2, \qquad (3)$$

of which qubits A_i (B_i) are in Alice's (Bob's) possession. The combined state $|t_0\rangle = |W_3\rangle_{123}|\mathcal{B}\rangle_{A_1B_1}|\mathcal{B}\rangle_{A_2B_2}$ of the total system can be expanded as

$$|t_0\rangle = \frac{1}{2}\times$$
$$\times[x_1(|1000000\rangle+|1000011\rangle+|1001100\rangle+|1001111\rangle)+$$
$$+x_2(|0100000\rangle+|0100011\rangle+|0101100\rangle+|0101111\rangle)+$$
$$+x_3(|0010000\rangle+|0010011\rangle+|0011100\rangle+$$
$$+|0011111\rangle)]_{123A_1B_1A_2B_2}. \qquad (4)$$

Our protocol proceeds in several steps as follows. **S1.** Alice performs 2 CNOT operations: a CNOT_{1A_1} on qubit-pair $(1, A_1)$ and another CNOT_{2A_2} on $(2, A_2)$, where $\mathrm{CNOT}_{ij}|a,b\rangle_{ij} = |a, a\oplus b\rangle_{ij}$ $\forall a,b \in \{0,1\}$ with $\oplus$ an addition mod 2. Accordingly, state $|t_0\rangle$ becomes $|t_1\rangle = \mathrm{CNOT}_{2A_2}\mathrm{CNOT}_{1A_1}|t_0\rangle$, which can be represented as

$$|t_1\rangle = \frac{1}{2}\times \qquad (5)$$
$$\times[|00\rangle_{A_1A_2}(x_1|10010\rangle+x_2|01001\rangle+x_3|00100\rangle)_{123B_1B_2}+$$
$$+|01\rangle_{A_1A_2}(x_1|10011\rangle+x_2|01000\rangle+x_3|00101\rangle)_{123B_1B_2}+$$
$$+|10\rangle_{A_1A_2}(x_1|10000\rangle+x_2|01011\rangle+x_3|00110\rangle)_{123B_1B_2}+$$
$$+|11\rangle_{A_1A_2}(x_1|10001\rangle+x_2|01010\rangle+x_3|00111\rangle)_{123B_1B_2}].$$

S2. Alice measures qubits A_1, A_2 in the z-basis $\{|0\rangle,|1\rangle\}$ with outcomes $\{l,m\} = \{0,0\}$, $\{0,1\}$, $\{1,0\}$ or $\{1,1\}$ if she finds $|00\rangle_{A_1A_2}$, $|01\rangle_{A_1A_2}$, $|10\rangle_{A_1A_2}$ or $|11\rangle_{A_1A_2}$, respectively.

S3. Alice publicly announces her measurement outcome for Bob to carry out the right action. Namely, if $\{l,m\} = \{0,0\}$, $\{0,1\}$, $\{1,0\}$ or $\{1,1\}$, Bob applies $(I\otimes I)_{B_1B_2}$, $(I\otimes\sigma_x)_{B_1B_2}$, $(\sigma_x\otimes I)_{B_1B_2}$ or $(\sigma_x\otimes\sigma_x)_{B_1B_2}$, respectively, on his qubits (B_1,B_2), where I is the unity operator and $\sigma_{x,y,z}$ are the Pauli operators. As a consequence, the state of the remaining five qubits 1, 2, 3, B_1 and B_2 transforms to $|t_2\rangle = (x_1|10010\rangle + x_2|01001\rangle + x_3|00100\rangle)_{123B_1B_2}$ which can also be rewritten in the x-basis $\{|\tilde{0}\rangle,|\tilde{1}\rangle\}$ of qubits $1,2,3$ as

$$|t_2\rangle = \left(\frac{1}{\sqrt{2}}\right)^3\times \qquad (6)$$
$$\times[(|\tilde{0}\tilde{0}\tilde{0} - |\tilde{1}\tilde{1}\tilde{1}\rangle)_{123}(x_1|10\rangle + x_2|01\rangle + x_3|00\rangle)_{B_1B_2} +$$
$$+(|\tilde{0}\tilde{0}\tilde{1} - |\tilde{1}\tilde{1}\tilde{0}\rangle)_{123}(x_1|10\rangle + x_2|01\rangle - x_3|00\rangle)_{B_1B_2} +$$
$$+(|\tilde{0}\tilde{1}\tilde{0} - |\tilde{1}\tilde{0}\tilde{1}\rangle)_{123}(x_1|10\rangle - x_2|01\rangle + x_3|00\rangle)_{B_1B_2} +$$
$$+(|\tilde{0}\tilde{1}\tilde{1} - |\tilde{1}\tilde{0}\tilde{0}\rangle)_{123}(x_1|10\rangle - x_2|01\rangle - x_3|00\rangle)_{B_1B_2}],$$

where $|\tilde{0}\rangle = (|0\rangle+|1\rangle)/\sqrt{2}$ and $|\tilde{1}\rangle = (|0\rangle-|1\rangle)/\sqrt{2}$.

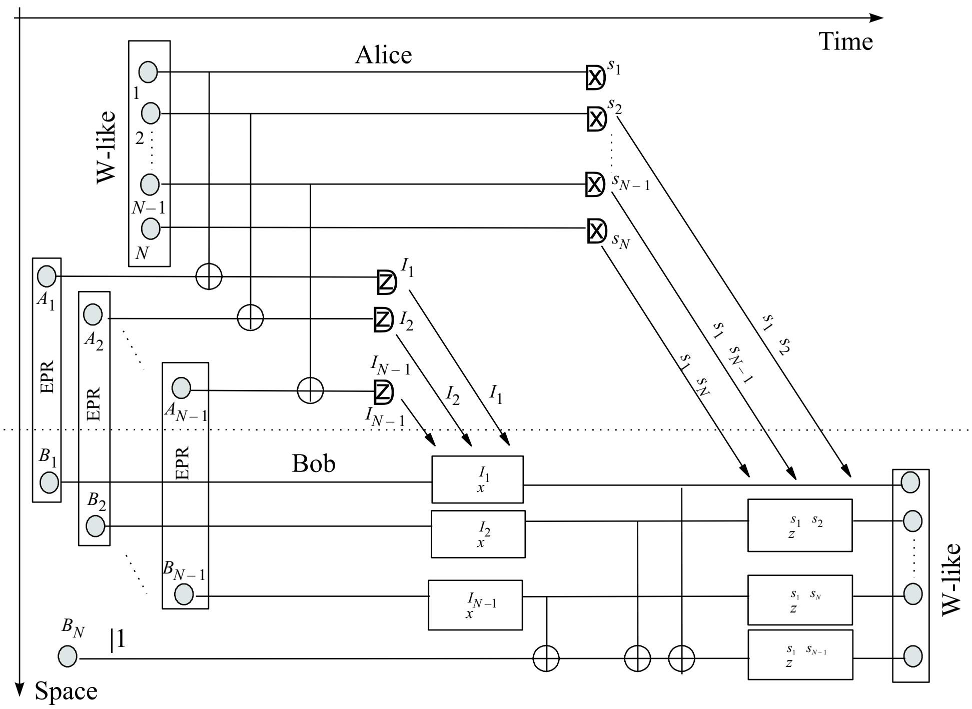
S4. Alice and Bob independently do the following. Alice measures her qubits $1,2,3$ in the x-basis, while Bob locally prepares an ancilla B_3 in state $|1\rangle_{B_3}$ and performs a $\mathrm{CNOT}_{B_2B_3}$ on his qubits (B_2,B_3) followed by another $\mathrm{CNOT}_{B_1B_3}$ on (B_1,B_3). As a result, state $|t_2\rangle$ transforms to $|t_3\rangle = \mathrm{CNOT}_{B_1B_3}\mathrm{CNOT}_{B_2B_3}|t_2\rangle$, which reads

$$|t_3\rangle = \left(\frac{1}{\sqrt{2}}\right)^3\times \qquad (7)$$
$$\times[(|\tilde{0}\tilde{0}\tilde{0}-|\tilde{1}\tilde{1}\tilde{1}\rangle)_{123}(x_1|100\rangle+x_2|010\rangle+x_3|001\rangle)_{B_1B_2B_3}+$$
$$+(|\tilde{0}\tilde{0}\tilde{1}-|\tilde{1}\tilde{1}\tilde{0}\rangle)_{123}(x_1|100\rangle+x_2|010\rangle-x_3|001\rangle)_{B_1B_2B_3}+$$
$$+(|\tilde{0}\tilde{1}\tilde{0}-|\tilde{1}\tilde{0}\tilde{1}\rangle)_{123}(x_1|100\rangle-x_2|010\rangle+x_3|001\rangle)_{B_1B_2B_3}+$$
$$+(|\tilde{0}\tilde{1}\tilde{1}-|\tilde{1}\tilde{0}\tilde{0}\rangle)_{123}(x_1|100\rangle-x_2|010\rangle-x_3|001\rangle)_{B_1B_2B_3}].$$

S5. Alice publicly broadcasts her measurement outcome for Bob to correctly reconstruct the state of his qubits (B_1,B_2,B_3) to be in the desired one. Denote by $\{i,j,k\}$ Alice's outcome corresponding to finding $|\tilde{i}\tilde{j}\tilde{k}\rangle_{123}$. At first glance, it follows from Eq. (8) that Bob will obtain, up to a global phase factor, the desired state by acting on (B_1,B_2,B_3) the operator $(\sigma_x^i\otimes\sigma_x^j\otimes\sigma_x^k)_{B_1B_2B_3}$. Nevertheless, a closer look at Eq. (8) verifies the simpler action as $(\sigma_x^{i\oplus j}\otimes\sigma_x^{i\oplus k})_{B_2B_3}$, i.e., Alice can publish just 2 bits $i\oplus j$ and $i\oplus k$ instead of 3 bits i, j and k of her full measurement outcomes, thus reducing the overall classical communication cost. This interesting feature, which is due to the specific structure of the W-like state, will be elucidated later when we deal with the general case of an arbitrary number N of qubits.

From above we see that to teleport a 3-qubit W-like state we used only 2 ebits plus 4 bits $(l, m, i\oplus j, i\oplus k)$and no necessity of BMs arose. This is clearly cheaper than that for teleportation of an arbitrarily general 3-qubit state which requires 3 ebits plus 6 bits as well as 3 BMs [10].

To make clearer and most explicit the general rules for the parties to follow we now turn to an arbitrary



Schematic illustration of teleportation of a unknown N-qubit W-like state. A qubit is represented by a solid circle. The arrows indicate classical communication: each arrow carries 1 bit. $X(Z)$ denotes measurement in the x-basis (z-basis) with the outcomes $\{s_1, s_2, ..., s_N\}$ ($\{l_1, l_2, ..., l_{N-1}\}$)

$N \geq 3$, i.e., we deal with the general N-qubit W-like state of the form (1) with unknown coefficients $x_1, x_2, \cdots, x_N$, except for $\sum_{n=1}^N |x_n|^2 = 1$. The particular structure of $|\mathrm{W}_N\rangle_{12\cdots N}$ allows us to represent it compactly as

$$|\mathrm{W}_N\rangle_{12\cdots N} = \sum_{a_1,\cdots,a_N=0}^{1} \delta_{a,1}\alpha_{a_1a_2\cdots a_N}|a_1, a_2, \cdots, a_N\rangle_{12\cdots N}, \tag{8}$$

where $a = \sum_{n=1}^N a_n$ and $\delta_{a,1}$ is the Kronecker symbol. In general, Alice and Bob have to share in advance $(N-1)$ identical EPR pairs in the form (3) with $i = 1, 2, \cdots, N-1$. The total system state $|T_0\rangle = |\mathrm{W}_N\rangle_{12\cdots N} \bigotimes_{j=1}^{N-1} |\mathcal{B}\rangle_{A_jB_j}$ can be written as

$$|T_0\rangle = \left(\frac{1}{\sqrt{2}}\right)^{N-1} \sum_{a_1,\cdots,a_N,b_1\cdots,b_{N-1}=0}^{1} \delta_{a,1}\alpha_{a_1a_2\cdots a_N} \times \left(\bigotimes_{m=1}^{N-1} |a_m, b_m\rangle_{mA_m}|b_m\rangle_{B_m}\right)|a_N\rangle_{A_N}. \tag{9}$$

Our general protocol can be implemented as follows (see Figure).

G1. Alice makes a CNOT_{mA_m} on a qubit-pair (m, A_m), for all $m = 1, 2, ..., N-1$, transforming $|T_0\rangle$ to $|T_1\rangle = \bigotimes_{m=1}^{N-1} \mathrm{CNOT}_{mA_m}|T_0\rangle$, i.e.,

$$|T_1\rangle = \left(\frac{1}{\sqrt{2}}\right)^{N-1} \sum_{a_1,\cdots,a_N,b_1\cdots,b_{N-1}=0}^{1} \delta_{a,1}\alpha_{a_1a_2\cdots a_N} \times \left(\bigotimes_{m=1}^{N-1} |a_m, a_m\oplus b_m\rangle_{mA_m}|b_m\rangle_{B_m}\right)|a_N\rangle_{A_N} = $$
$$= \left(\frac{1}{\sqrt{2}}\right)^{N-1} \sum_{a_1,\cdots,a_N,l_1\cdots,l_{N-1}=0}^{1} \delta_{a,1}\alpha_{a_1a_2\cdots a_N} \times \bigotimes_{m=1}^{N} |a_m\rangle_m \left(\bigotimes_{j=1}^{N-1} |l_j\rangle_{A_j}|a_j\oplus l_j\rangle_{B_j}\right). \tag{10}$$

G2. Alice measures her qubits $A_1, A_2, \ldots, A_{N-1}$ in the z-basis with corresponding outcomes $\{l_1, l_2, \cdots, l_{N-1}\}$, if she finds $\otimes_{j=1}^{N-1}|l_j\rangle_{A_j}$, projecting $|T_1\rangle$ onto

$$|T_1'\rangle = \sum_{a_1,\cdots,a_N=0}^{1} \delta_{a,1}\alpha_{a_1a_2\cdots a_N} \bigotimes_{m=1}^{N} |a_m\rangle_m \bigotimes_{j=1}^{N-1} |a_j\oplus l_j\rangle_{B_j}.$$

G3. Alice announces her measurement outcomes $\{l_j\}$, $j = 1, 2, ..., N-1$. Because of the structure of $|T_1'\rangle$ and the property $\sigma_x^l|a\rangle = |a\oplus l\rangle$ $\forall a, l \in \{0, 1\}$, Bob, after hearing Alice's announcement, is able to cast $|T_1'\rangle$ to $|T_2\rangle = \sum_{a_1,\cdots,a_N=0}^{1} \delta_{a,1}\alpha_{a_1a_2\cdots a_N} \bigotimes_{m=1}^{N} |a_m\rangle_m \bigotimes_{j=1}^{N-1} |a_j\rangle_{B_j}$

by acting $\sigma_x^{l_j}$ on his qubits B_j, for all $j = 1, 2, ..., N-1$. $|T_2\rangle$ can also be rewritten in the x-basis of qubits $\{m\}$ as

$$|T_2\rangle = \left(\frac{1}{\sqrt{2}}\right)^N \times$$
$$\times \sum_{a_1,\cdots,a_N,s_1,\cdots,s_N=0}^{1} \delta_{a,1}\alpha_{a_1a_2\cdots a_N}(-1)^{a_1s_1\oplus\cdots\oplus a_Ns_N}\times$$
$$\times \bigotimes_{m=1}^{N} |\widetilde{s_m}\rangle_m \bigotimes_{j=1}^{N-1} |a_j\rangle_{B_j}, \tag{11}$$

where $|\tilde{s}\rangle = \sum_{a=0}^{1}(-1)^{as}|a\rangle/\sqrt{2}$ and the equality $(-1)^{a_1s_1+\cdots+a_Ns_N} \equiv (-1)^{a_1s_1\oplus\cdots\oplus a_Ns_N}$ has been used.

G4. Alice measures all her qubits $\{m\}$ in the x-basis with outcomes $\{s_m\} = \{s_1, s_2, \cdots, s_N\}$ corresponding to finding $\otimes_{m=1}^{N}|\widetilde{s_m}\rangle_m$, while Bob independently prepares an ancilla B_N in state $|1\rangle_{B_N}$ and makes a $\text{CNOT}_{B_{N-1}B_N}$ on qubits (B_{N-1}, B_N) followed by a sequence of $(N-2)$ $\text{CNOT}_{B_jB_N}$ on (B_j, B_N) with $j = 1, 2, \cdots, N-2$. After such actions of Alice and Bob, state (11) becomes

$$|T_3\rangle = \sum_{a_1,\cdots,a_N=0}^{1} \delta_{a,1}\alpha_{a_1a_2\cdots a_N}(-1)^{a_1s_1\oplus\cdots\oplus a_Ns_N}\times$$
$$\times \bigotimes_{j=1}^{N-1} |a_j\rangle_{B_j}|a_1\oplus\cdots\oplus a_{N-1}\oplus 1\rangle_{B_N}. \tag{12}$$

From the constraint $a = \sum_{n=1}^{N} a_n = 1$, which is due to the specific structure of the W-like state, two equalities can be derived. The first one is

$$a_1 \oplus \cdots \oplus a_{N-1} \oplus 1 = a_N \tag{13}$$

and the second one is

$$a_1s_1\oplus\cdots\oplus a_Ns_N = s_1\oplus(s_1\oplus s_2)a_2\oplus\cdots\oplus(s_1\oplus s_N)a_N. \tag{14}$$

Substituting (13) and (14) into (12) yields

$$|T_3\rangle = (-1)^{s_1}\times$$
$$\times \sum_{a_1,\cdots,a_N=0}^{1} \delta_{a,1}\alpha_{a_1a_2\cdots a_N}(-1)^{(s_1\oplus s_2)a_2\oplus\cdots\oplus(s_1\oplus s_N)a_N}\times$$
$$\times \bigotimes_{n=1}^{N} |a_n\rangle_{B_n}. \tag{15}$$

G5. Alice broadcasts her measurement outcome in terms of $(N-1)$ bits in the form $\{(s_1\oplus s_2), (s_1\oplus s_3), \cdots, (s_1\oplus s_N)\}$ and Bob, after hearing Alice's announcement, applies a single-qubit operation $\sigma_z^{s_1\oplus s_n}$ on each of his qubit B_n, for all $n = 2, 3, \cdots, N$. Because $\sigma_z^s|a\rangle = (-1)^{sa}|a\rangle\ \forall a, s \in \{0, 1\}$, state (15) is converted into

$$|T_4\rangle = (-1)^{s_1}\times$$
$$\times \sum_{a_1,\cdots,a_N=0}^{1} \delta_{a,1}\alpha_{a_1a_2\cdots a_N}|a_1, a_2, \cdots, a_N\rangle_{B_1B_2\cdots B_N} \tag{16}$$

which, up to a global phase factor $(-1)^{s_1}$, is nothing else but the desired N-qubit W-like state, now appears at Bob's location among the qubits B_1, B_2, ..., and B_N. It is worthy emphasizing an interesting feature that directly from Eq. (15) Bob could simply apply $\sigma_z^{s_n}$ on each qubit B_n, for all $n = 1, 2, \cdots, N$, consuming thus N bits from Alice. However, the equality (14) allows one to safe 1 bit as detailed above.

In summary, we have presented a protocol to teleport a unknown N-qubit W-like state with an arbitrary $N \geq 3$. Compared to the well-known universal protocol for teleportation of a general N-qubit state which requires N ebits plus $2N$ bits together with N BMs, our protocol is more economical since it consumes just $(N-1)$ ebits and $2(N-1)$ bits $\{l_1, \cdots, l_{N-1}, s_1\oplus s_2, \cdots, s_1\oplus s_N\}$. It also has a technical advantage in the sense that no BMs are necessary at all (at the expense of a sequence of CNOTs at both sending and receiving stations). Though teleportation protocols in QED can also be done without BMs [13], in most cases their success probability cannot exceed 50%. Here both unit success probability and unit fidelity are achieved, even without BMs. Thus, our protocol would be of broad interest since it is economical and feasible within current technologies.

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