Doubly logarithmic asymptotic behavior of exclusive cross sections in quantum chromodynamics

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The process of single-photon e^+e^- annihilation into a quark-antiquark pair and into an arbitrary number of gluons, which has a simple stochastic explanation, is given in a doubly logarithmic approximation.

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It is well known that many perturbation theories for the amplitudes of the processes in quantum chromodynamics at large momentum transfer contain double logarithms. These double logarithms in the inclusive cross sections are canceled (fully or partially) by the double logarithms that appear in the integration over the phase volume of nonrecordable particles (see, for example, review article of Ref. 1.). The more comprehensive the description of the processes, the greater is the role played by the double logarithms and the greater the importance attributed to the doubly logarithmic asymptotic behavior of the amplitudes of the processes with an arbitrary number of

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particles and to the regions of phase volume, which lead to a doubly logarithmic contribution to the cross section.

An exclusive description of the process of single-photon e^+e^- annihilation to a quark-antiquark pair and to an arbitrary number of gluons is given below in a doubly logarithmic approximation. Since the analysis of the process is rather complicated and cumbersome, we shall skip it here; however, since the result presented below is surprisingly simple and physically obvious, we can generalize it to include other processes.

We shall analyze the creation of a quark-antiquark pair and of n gluons initially in the Born approximation. A characteristic feature of quantum chromodynamics, which violates the Poisson distribution of the radiation,[2] is the decay of a gluon into two gluons. Because of this, an attempt to write a single expression for the matrix element, just as in quantum electrodynamics, gives rise to enormous expressions for $n = 3$. It is more convenient to isolate a set of nonoverlapping regions, each of which has a matrix element expressed in a simple form. It is also more convenient to approach this problem in terms of angles and energies rather than Sudakov variables that are customarily used in quantum electrodynamics, and use the physical polarization vectors of gluons. Suppose that $p_-(p_+)$ is the 4-momentum of a quark (antiquark) and k_i is the 4-momentum of the ith gluon; $i = 1,2,...n$ and ϵ_-,ϵ_+, and ω_i are the corresponding energies in the c.m. of the nascent system. Only the regions in which the gluon energies are highly ordered contribute; moreover, the gluons are soft; for specificity, we assume that

$$\omega_n \ll \omega_{n-1} \ll \ldots \ll \omega_1 \ll \epsilon_\pm \approx \epsilon . \tag{1}$$

We shall examine an ordinary Feynman diagram without the four-gluon vertices. Because of strong ordering (1), each virtual particle in the diagram has an energy which is approximately equal to that of the most energetic particle of those final particles into which it decays. We shall assign to the ith gluon the line of the virtual particle with an energy which is approximately equal to ω_i. If the line of the ith gluon begins with the jth gluon line (which is possible only when $i>j$), then we can say that the jth gluon emits the ith gluon; if this line begins with the quark (antiquark) line, then the ith gluon is emitted as a quark (antiquark). We shall compare a certain kinematic region with this diagram. The particle energies in this region satisfy the condition (1) and the region can be expressed with respect to the angles in the following way. If the ith gluon, which is emitted by the jth particle, emits gluons $i_1, i_2,...i_r$ successively along its line, then

$$\widehat{\mathbf{k}_i\mathbf{k}_j} \gg \widehat{\mathbf{k}_i\mathbf{k}_{i_1}} \gg \widehat{\mathbf{k}_i\mathbf{k}_{i_2}} \gg \ldots \gg \widehat{\mathbf{k}_k\mathbf{k}_{ir}} , \tag{2}$$

where $\widehat{\mathbf{k}_i\mathbf{k}_j}$ is the angle between the $\mathbf{k}_i$ and $\mathbf{k}_j$ vectors. If the ith gluon is emitted by a quark (antiquark), then Eq. (2) has $\mathbf{p}_-(\mathbf{p}_+)$ instead of $\mathbf{k}_j$. The angles are ordered for the gluons $j_1,...j_m$, which are emitted successively by a quark (antiquark) in the direction of (or opposite to) the quark line

$$1 \gg \widehat{\mathbf{p}_\mp\mathbf{k}_{j_1}} \gg \ldots \widehat{\mathbf{p}_\mp\mathbf{k}_{j_m}} . \tag{3}$$

The matrix element of the process in the kinematic region defined by the inequalities

(1)–(3) is

$$M_0 g^n (-1)^m \prod_{i=1}^{n} \frac{(e_i P_i)}{(k_i P_i)} G. \qquad (4)$$

Here M_0 is the matrix element of the creation of a quark-antiquark pair without its group part, m is the number of gluons emitted by an antiquark, e_i is the polarization vector of the jth gluon, P_i is the 4-momentum of the particle that emitted it, G is the group part of the matrix element, which is obtained if the quark-line vertices in the Feynman diagram, which is compared with the examined kinematic region, have $\lambda^a/2$ and the three-gluon vertices have if^{abc}, where $a(b)$ is the superscript of the gluon with the smallest (largest) energy.

Thus, each Feynman diagram without four-gluon vertices is compared with the kinematic region which is determined by the inequalities (1)–(3). The matrix element of the process in the Born approximation in this region has the form (4). We should note that the expression (4) was obtained by summing the contributions of the Feynman diagrams; a gauge in which only one diagram contributes does not exist.

If the virtual corrections are taken into account, then Eq. (4) must be multiplied by the doubly logarithmic form factor F (F was calculated in Ref. 3 for the process without emission of gluons)

$$F = \exp\left[-\frac{1}{2} w^F_{p_-}(1) - \frac{1}{2} w^F_{p_+}(1) - \frac{1}{2} \sum_{i=1}^{n} w^V_{k_i}(\theta_i)\right], \qquad (5)$$

where θ_i is the angle between the momenta of the ith gluon and the particle that emitted it and $w_p^{F(V)}(\theta)$ is the Born probability of the emission of a gluon by a fermion (gluon) with the momentum p into a cone with aperture angle θ

$$w_p^{F(V)}(\theta) = \frac{g^2}{(2\pi)^3} C_{F(V)} \int \frac{d^3k}{2\omega} \frac{\mathbf{p}^2\theta_k^2}{(kp)^2}. \qquad (6)$$

Here θ_k is the angle between $\mathbf{p}$ and $\mathbf{k}$; the integration over θ_k is carried out to θ and the integration over ω is carried out to p°.

The contribution to the cross section summed over the spin and color states of the nascent particles is

$$\sigma_0 \int F^2 \prod_{i=1}^{n} \frac{g^2}{(2\pi)^3} C_i \frac{d^3k_i}{2\omega_i} \frac{\mathbf{P}_i^2 \theta_{k_i}^2}{(k_i P_i)^2} \qquad (7)$$

where σ_0 is the production cross section of a quark-antiquark pair, θ_{k_i} is the angle between $\mathbf{k}_i$ and $\mathbf{P}_i$, $C_i = C_F(C_V)$ if the gluon is emitted by a quark (gluon), $C_F = \frac{N^2-1}{2N} = \frac{4}{3}$, and $C_V = N = 3$. The integration range in Eq. (7) is limited by the inequalities (1)–(3). The sum of all these regions is equal to the total cross section.

It should be noted that, as follows from Eqs. (4)–(7), the emission of gluons in each region is essentially a classical emission in the doubly logarithmic approximation. Equation (7) evidently allows an interpretation in terms of decay probabilities, which allows us to use the "parton" language in the analysis of the process (see Ref. 4).

This picture can easily be tested for self-consistency: if we calculate the total cross section summed over the number of gluons n from 0 to ∞, then the double logarithms cancel out.

The integrals must be regularized in the specific calculations in Eqs. (6) and (7).

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