

Difference in the behavior of the structure functions of annihilation and of deep inelastic scattering

M. G. Ryskin and Yu. L. Dokshitser

B. P. Konstantinov Institute of Nuclear Physics, USSR Academy of Sciences

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The causes of violation of the Gribov-Lipatov relation

$D_{eN}(x, Q^2) = \bar{D}_{e^+e^-}^B(x, Q^2)$ in the region of small x are analyzed. At $x \rightarrow x_{\min} \sim (\sqrt{q^2}/Q^2) D_{eN} \gg \bar{D}_{e^+e^-}^B$.

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As is well known, the structure functions of deep inelastic scattering — $D_A^B(x, q^2)$ (which describe the probability of finding the B parton in the A parton with a virtuality q^2 , which carries a fraction x of the momentum of the A particle) coincide with those of e^+e^- annihilation — $\bar{D}_A^B(x, q^2)$ (which describe the probability of finding the B parton with a fraction x of the momentum in the A parton which has a virtuality q^2) in the region of relatively large ($x \sim 1$) Bjorken variables^{1,2}

$$\bar{D}_A^{-B}(x, Q^2) = D_A^B(x, Q^2). \quad (1)$$

This equivalence, which corresponds to the Gribov-Lipatov relation,³ was demonstrat-

ed within the context of the main logarithmic approximation (MLA) of the field theory, in which each degree of the weak coupling constant α_s is balanced by the large virtuality logarithm $\ln q^2$.

We show in this paper that the relation (1) is not valid in the region of small $x(\ll 1)(\alpha_s \ln x^{-1} \gtrsim 1)$ and analyze the kinematic reasons for the difference in the behavior of the structure functions of e^+e^- annihilation and of the deep inelastic $[e(\nu)N]$ processes.

1) The first reason is that the target partons collide with the protons of the virtual photon (Z, W boson) in $e(\nu)N$ collisions. Therefore, the target partons that move in one direction stick together and shade each other at $x \ll 1$ when the parton density xD increases sharply.

The annihilation, however, involves a series decay of heavy partons (with a positive virtuality q_i^2). As a result of each decay ($q_i \rightarrow q_{i+1} + q$), the newly produced particles, which fly apart (in the rest frame q_i) in different directions, have virtually no time to interact again.

Thus, a whole class of diagrams corresponding to parton rescattering processes is missing in the annihilation [in comparison with $e(\nu)N$]. We shall prove this formally.

A. In addition to the conventional ladder diagrams in Fig. 1(a), whose structure is the same in e^+e^- and $e(\nu)N$ collisions, the diagram in Fig. 1(b), which corresponds to Reggeization of a gluon with a momentum q_2 , contributes significantly ($\sim \alpha_s \ln x^{-1}$) to deep inelastic scattering (Ref. 4).¹⁾ The $\ln x^{-1}$ coefficient in the Sudakov variables⁵ ($q_\mu = \alpha p_{B\mu} + \beta p_{A\mu} + q_{i\mu}$) corresponds to the range of integration over $\alpha': \tilde{\alpha}_i \sim \alpha_1 \gg \alpha' \gg \alpha_2 \sim \tilde{\alpha}_2$.

Since the β_i variables in the $e(\nu)N$ case run in the opposite direction to $\alpha_i (\tilde{\beta}_2 \gg -\beta_2 \sim \tilde{\beta}_1 \gg -\beta_1)$, the virtualities $\tilde{q}_2^2 \approx \tilde{q}_{2i}^2$ and $q'^2 \approx q_i'^2$ are determined solely by the transverse momenta q_{2i} and q'_i and the value of α' (after the path of β' is closed at the pole $1/\tilde{q}_1^1$) turns out to be important only in the propagator $1/\tilde{q}_2^2 \approx x/\alpha' \tilde{\beta}_2 Q^2$, which gives the logarithmic integral $d\alpha'/\alpha'$.

Both α_i and β_i in the annihilation flow in the same direction²⁾ (from the virtual photon). The value of $\beta_2 \sim \tilde{\beta}_2$ is now large. As a result, we obtain one more large propagator $1/q_2^2 \approx 1/a' \beta_2 Q^2 \approx 1/\tilde{q}_2^2$, which breaks down the logarithmic integration

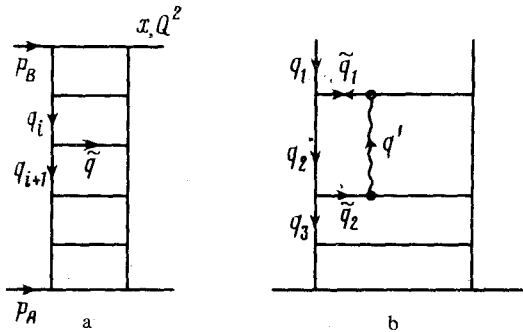


FIG. 1.

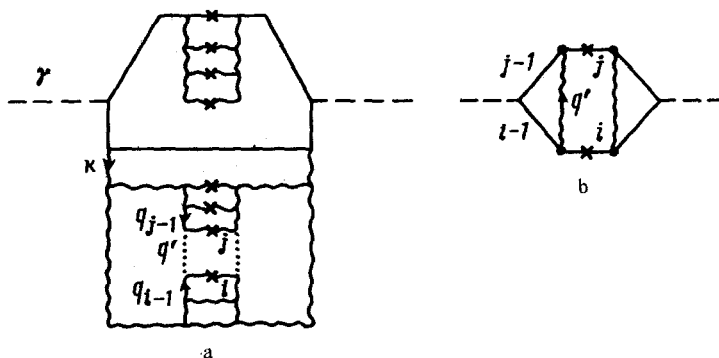


FIG. 2.

with respect to $d\alpha'$. Thus, we can see that the contribution of the diagram in Fig. 1(b) in the e^+e^- case is proportional to the coupling constant $\alpha_s(q_2^2)$, i.e., it is negligible. This small value reflects the decay kinematics. The factor $(s_{12}/M^2)^{\alpha_s}$ where $s_{12} = (\vec{q}_1 + \vec{q}_2)^2$, corresponds to Reggeization of the q_2 gluon and the largest mass q_1^2 must be selected as the mass M^2 . If the ratio $s_{12}/q_1^2 \sim 1/x$ in $e(\nu)N$, then $s_{12}/q_1^2 \lesssim 1$ in e^+e^- and $s_{12}/q_1^2 \alpha_s \sim \alpha_s \ln(s_{12}/q_1^2) \sim \alpha_s$.

B. Diagram 1(b) describes the interaction of two nearest partons. A question arises whether the rescattering probability of a pair of more distant partons w is nonetheless large, and whether it has excessive parton density $\tilde{D}(w \propto \alpha_s \tilde{D}?)$.

We shall show now that this does not occur. In fact, any two partons (i and j in Fig. 2), which are produced as a result of decay of a heavy k parton, fly apart in different directions in its rest frame. An interaction between them can produce only a light q' gluon which is emitted backward relative to the direction of the i jet. As we have seen in Fig. 1(b), the probability of such emission does not have large logarithms ($\ln x$ or $\ln Q^2$) and is proportional to $\alpha_s \ll 1$. The probability of capture of a q' gluon by a j jet (more precisely, by the entire group of particles situated (Fig. 2) above the i parton recorded by us) cannot exceed unity even when $\tilde{D} \rightarrow \infty$. Therefore, the contribution of any rescattering of partons in the annihilation is $\lesssim \alpha_s \ll 1$.

Since the energy of q' is much lower than that of i and j partons and since it does not carry any quantum numbers (except color), its exchange virtually does not change the inclusive distributions. This can be formally demonstrated by the fact that their mutual contributions decrease [this can easily be verified by using the diagram in Fig. 2(b)] as a result of closing the path of integration of β' at the poles $1/q_{i-1}^2$ and $1/q_{j-1}^2$ (etc.).

Thus, only the ladder diagrams in Fig. 1(a) remain in the annihilation [in contrast with $e(\nu)N$] when x are small.

2) The decay kinematics of e^+e^- , however, create an additional complication. If the momentum ratio α_{i+1}/α_i varies from 0 to unity in the deep inelastic case in which one parton has a negative virtuality $q_i^2 < 0$ at each stage, then a purely kinematic constraint $\alpha_{i+1}/\alpha_i \geq q_{i+1}^2/q_i^2$ can occur in the annihilation in which all the masses are

positive. Therefore, the decay probabilities, i.e., the kernels of the relevant Bethe-Salpeter equation,^{2,6} by means of which the ladder diagrams in Fig. 1(a) are summed, must be multiplied by $\theta\left(\frac{\alpha_{i+1}}{a_i} - q_{i+1}^2/q_i^2\right)$. Solving the obtained equation (by using the method described elsewhere³), we obtain, in the region of small x ,

$$\bar{D}(x, Q^2, q_0^2) \propto \exp \left[4 \sqrt{\frac{N}{\beta_2}} \left(\sqrt{\ln Q^2} - \sqrt{\ln q_0^2} \right) - \sqrt{\frac{9N}{2\beta_2}} (\gamma - \ln x^{-1})^2 / \gamma^{3/2} \right], \quad (2)$$

where $N=3$ is the number of colors, $\beta_2 = 11N/3 - 2n_F/3$ is the leading coefficient of the Gell-Mann-Low function, q_0^2 is the virtuality of the recorded parton, and $\gamma = \frac{1}{2}(\ln Q^2 - \ln q_0^2)$ is the plateau width.

We emphasize that the \bar{D} function differs substantially from the structure function of deep inelastic scattering. As shown in Ref. 8, $\ln x \sim \frac{1}{2} \ln Q^2$ in the same region

$$D(x, Q^2) \propto \exp \left[4 \sqrt{\frac{N}{\beta_2}} \ln x^{-1} \ln \frac{\ln Q^2}{\sqrt{\ln 1/x}} \right] \gg \bar{D}(x, Q^2).$$

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¹For specificity, we use here the Feynman gauge.

²We selected the momentum \mathbf{p}_B in the direction of the B parton, $p_{B0} = P_{A0} = |\mathbf{p}_B| = (\sqrt{Q^2}/2; p_A = -\mathbf{p}_B$.

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