Nonlinear waves at the surface of a liquid metal in an electric field

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A panoramic electron micrograph of nonlinear capillary waves excited by an electric field at the surface of a liquid metal and "frozen in place" by rapidly cooling the metal has been obtained. The wave structure is compared with theoretical predictions.

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Many years ago, Tonks and Frenkel'[1-3] derived a theory for the instability of the surface of a liquid metal in an electric field directed normal to the surface, but there has been no experimental study of the structure of the nonlinear waves excited under these conditions. An understanding of this structure would be quite useful, particularly in connection with the development of liquid-metal ion emitters operating at a current density $\sim 10^4$ A/cm^2.[4]

In this letter we are describing a method for studying this instability. When a metal is rapidly solidified, the surface relief is "frozen in place." This relief is determined by the electric field E and by the surface tension α, whose effect is directed opposite that of the field. When this surface is examined in a scanning electron microscope it is possible to obtain a complete panoramic view of the "frozen" nonlinear capillary waves and also views of regions of particular interest. This method has finally made it possible to compare the nonlinear waves excited experimentally with the corresponding theory. More importantly, this method yields information required for deriving a theory for the emission of charged particles from unstable formations of this type.

The metal surface was melted, and a strong electric field was produced near the surface, by a stream of a dense, highly ionized hydrogen plasma with an initial radius ~ 1 mm. The plasma was transported in a vacuum by a strong magnetic field to the surface of a copper plate, which was held at a negative potential ϕ with respect to the plasma and which served as an ion collector. At an axial ion current density $j_+ = 10$ A/cm^2 and at a potential $\phi = 400$–500 V, the specific ion bombardment power was (4-5)$\times 10^3$ W/cm^2 in cw operation. The calculated value of the electric field at the collector surface, which was separated from the plasma by a plane ion space-charge sheath, was $E=(16\pi j_+)^{1/2}(2e/m)^{-1/4}\phi^{1/4}\approx 10^5$ V/cm.

In order to excite unstable capillary waves, the characteristics of the ion beam and the heat removal from the copper plate must be chosen in such away that at least part of the surface is liquid and is in a strong electric field. Calculations show, however, that these waves can be frozen after the discharge producing the plasma is turned off only if this liquid is very shallow; otherwise, the capillary waves will decay before the metal solidifies immediately after the discharge is turned off. The conditions required for freezing the waves in place were established as follows: The plate was subjected to ion bombardment, during which primarily the central part of the plate melted. Because of the resultant momentum associated with the pressure of the plasma electrons, liquid metal was forced away from the center of the plate toward its periphery. Convincing evidence of this radial displacement of liquid came from the solidified copper ridge extending above the original plane of the plate. Figure 1a shows the crater produced by the ion bombardment; here z is the direction along the ion beam. A scanning electron microscope with a resolution of 100 Å produced the

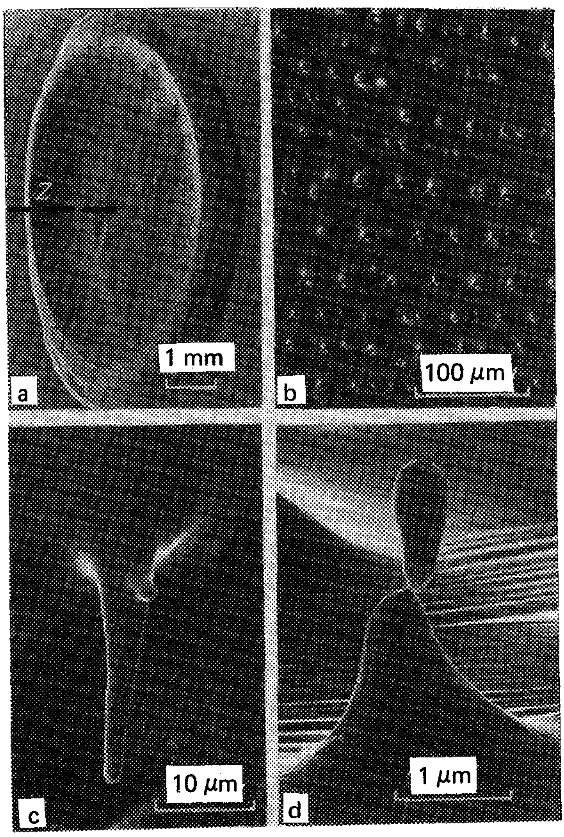

FIG. 1.

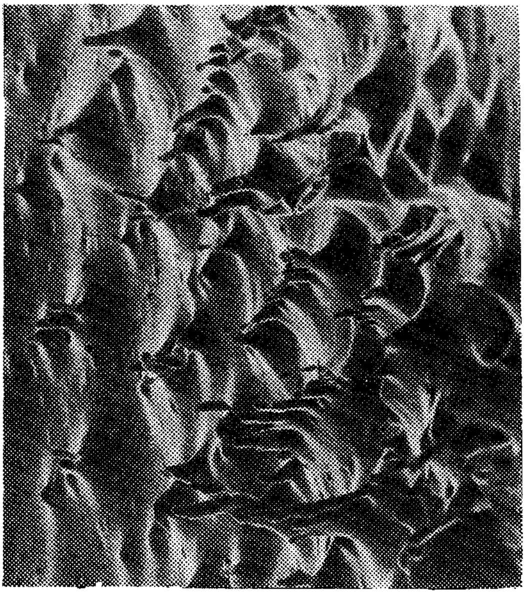

FIG. 2.

sharpest panoramic view of frozen nonlinear capillary waves in the "shallows" at the periphery of the crater, in an annulus bounded by the radii $r_1 = 2$ mm and $r_2 = 3$ mm (Fig. 2). The same waves are shown in plan view in Fig. 1b, from which we can determine the wavelength. Let us examine and interpret the results.

1. In an electric field, capillary waves are described by the dispersion relation[3] $\omega^2 = k/\rho\ (ak^2 - E^2/4\pi k + \rho g)$, which refers to a system in which the fixed electrode is quite far from the surface of the liquid metal. Under our experimental conditions, this fixed electrode was the plasma boundary. In the case of long-wavelength perturbations at the surface of a liquid metal, this plasma boundary follows the perturbations, since the boundary is separated from the surface by an ion sheath of fixed thickness $d_{sh} = (9\pi)^{-1/2}\ (2\ e/m)^{1/4}\ \phi^{3/4}\ j_+^{-1/2} \approx 100\ \mu$m. Under these conditions, perturbations with $\lambda >> d_{sh}$ cannot grow, because the distribution of E over the metal surface is conserved. We should expect wave buildup only in the case $\lambda \approx d_{sh}$, and this condition was actually met in the present experiments: $\lambda_{expt} = 50\ \mu$m.

2. The relatively large value of $k_{expt} = 2\pi/\lambda_{expt}$ and the vertical arrangement of the plate allow us to ignore the term ρg in the dispersion relation. The critical field required for exciting capillary waves is thus $E_{cr} = (4\pi\alpha k_{expt})^{1/2} \approx$ V/cm. This result is again consistent with the experimental results, since the actual field E in the case of a short-wavelength perturbation with a large initial amplitude is much higher than that calculated for a plane sheath. On the other hand, the large amplitude initial short-wavelength perturbation, which is required, is produced by the observed oscillations of the plasma in the source.

3. The instability in the Tonks model occurs in two stages. The first stage ends with the formation of a roughly hemispherical protuberance on the surface of the li-

quid metal. In the second stage, a peak with decreasing radius of curvature grows from the top of this hemisphere. The ratio of heights reached in the first and second stages is no greater than three.

According to Fig. 2, the shape of the protuberances at the wave crests agrees in general with the predictions of the theory: We can clearly see these two stages. Shown separately in Fig. 1c is a peak with a radius of curvature $R = 1\ \mu m$ at its tip which developed in the second stage. According to Tonks, the maximum peak height is $h_{max} = 1.35 \cdot 8\pi/E^2$, or $h_{max}/\lambda = 0.4$ at the critical field. The height of the fully developed peak shown in Fig. 1c agrees satisfactorily with the value $h_{max} = 0.4\lambda_{expt} = 20\ \mu m$. Equating the pressure caused by the surface tension to that caused by the electric field, we find the field at the tip of the peak to be $E_t \approx 6 \times 10^6$ V/cm. A field of the same order of magnitude can be found directly from the estimate $E_t \approx \phi/R$.

4. Many of the peaks in the micrograph have necks. Figure 1d shows one such case, for a peak that was frozen before the tip broke off. several other micrographs show that at least some of the peaks are definitely hollow.

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