Rossby soliton

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A Rossby soliton, which is a geostrophic eddy—an anticyclone, has been detected experimentally for the first time. The soliton is formed in a layer of "shallow water", which is rotating in a parabolic container (together with the container), and it moves opposite to the rotation, preserving its shape—a constant altitude profile. The ease with which the soliton is excited—despite considerable viscosity and friction—gives a basis for favoring the current theoretical model,[4] according to which the great red spot of Jupiter is a Rossby soliton.

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Rossby waves, which appear in the atmosphere (or ocean) of a rotating planet because of the spatial nonuniformity (latitude dependence) of the Coriolis force,[1] are well known in atmospheric physics. In terms of their characteristics these waves are analogous, to a large degree, to drift waves in a magnetized plasma with an inhomogeneous density[2] or temperature.[3] It was shown in a recent theoretical paper[4] that Rossby waves can form solitons that are like plasma drift solitons, and a model was proposed, according to which the famous red spot of Jupiter is a Rossby soliton. Our work was undertaken in order to verify this model experimentally.

In organizing the experiment we started from the need to verify the basic conclusions of the theory,[4] which consist of the following.

1. The Rossby soliton is a "local" eddy in a shallow layer of rotating fluid whose depth is small compared with the dimensions of the eddy. The eddy axis is normal to the fluid surface at the center of the eddy. The fluid height is greater inside a Rossby eddy than outside it. The eddy rotates in a direction opposite to the global rotation of the fluid and is a "mound" on its surface. In other words, it is an anticyclone, in which so-called geostrophic equilibrium is realized: the excess of hydrostatic pressure is balanced by the Coriolis force, which acts on the circular stream of particles around the eddy axis.[1]

2. The Rossby soliton moves (drifts) in the latitude direction, opposite to the global rotation of the fluid. The shape of the soliton—the fluid height profile—is preserved in this motion: the dispersion spreading of the eddy is compensated for by the nonlinearity. The soliton moves with a velocity that is slightly greater than the phase velocity of the *linear* Rossby wave, which is defined by the following relation:

$$v_{\text{ph}} = \frac{gH_0 \sin\alpha_0}{2\omega_0 R \cos^2\alpha_0}, \tag{1}$$

where g is the gravitational acceleration, H_0 is the unperturbed fluid height, $\omega_0 = 2\pi f_0$ is the angular velocity of its global rotation, R is the radius of curvature of the fluid surface in its meridional cross section at the center of the eddy, and α_0 is the angle between the axis of the global rotation of the fluid and the normal to its surface.

3. The characteristic size (diameter) $2a$ of the Rossby soliton is defined by the relation

$$2a \approx 3{,}5\,|r_0|\,h^{-1/2}, \tag{2}$$

where $h = H/H_0 - 1$ is the relative amplitude of the soliton, H is the fluid height in the soliton, and $r_0 = (gH_0)^{1/2}/2\omega_0 \cos\alpha_0$ is the Rossby radius.

The quantity h is a small parameter in the theory; nevertheless, we can make a qualitative comparison of the theory with experiment even as $h \to 1$.

As suggested by Petviashvili,[4] we chose "shallow water" of approximately constant depth in a container with a parabolic bottom, which was rotated about the vertical symmetry axis, as the object for excitation of the Rossby soliton (Fig. 1); the maximum inside diameter of the paraboloid was 28 cm. At a rotation frequency $f_0 = 1.8$ Hz the water covered the container bottom in a uniform layer with a depth $H_0 \approx 0.3$–0.5 cm. The shape of the water surface in the meridional cross section, i.e., in the cross section defined by a vertical plane passing through the axis, corresponded to the equation.

$$gz = \frac{1}{2}\omega_0^2 r^2 \quad \text{or} \quad z = 6 \cdot 10^{-2} r^2, \tag{3}$$

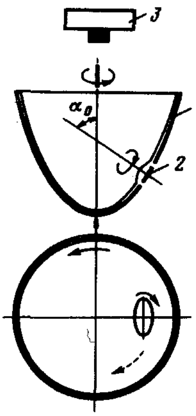

FIG. 1. Schematic of experimental setup: 1, container with parabolic bottom profile; 2, rotating disk; 3, movie camera. The dashed arrow in the top view shows the direction of Rossby-soliton drift—the soliton lags behind the global motion of the fluid.

where z and r are the vertical and horizontal coordinates of a point on the fluid surface. The hydrostatic pressure of the liquid in such a model is determined by the resultant gravitational force and by the centrifugal force of the fluid's rotation. The spatial nonuniformity of the acceleration $g/\cos\alpha$ of the resultant force alters the phase velocity of the linear Rossby wave:

$$v_{ph} = \frac{gH_0 \sin\alpha_0}{\omega_0 R \cos^3\alpha_0} = H_0\,\omega_0 \sin\alpha_0 \tag{4}$$

[Eq. (3) and the well-known expression for R have been used here]. The velocity of the Rossby eddy exceeds v_{ph}. Accordingly, we must substitute $g \to g/\cos\alpha_0$ in the formula for r_0.

The point with the coordinates $r = 10$ cm and $z = 6$ cm, at which $\cos\alpha_0 \approx 0.6$, the radius of curvature of the liquid surface in the meridional cross section [see Eq. (3)] is $R \approx 32$ cm, and the Rossby radius at $H_0 = 0.3$ cm and $f_0 = 1.8$ Hz is $r_0 \approx 1.7$ cm, was chosen for excitation of the Rossby eddy. A thin metal disk with a diameter of 3 cm was rotated about this point near the bottom of the liquid. The disk gradually entrained the water above it and in its vicinity into a local rotation.

The tests showed that for disk rotation in the "necessary" direction, i.e., opposite to the global rotation of the container (Fig. 1), a geostrophically equilibrium anticyclonic eddy appears above the disk—a mound of water, whose height ΔH is determined (and is adjustable) by the frequency and duration of the disk rotation, is set into rotation. A typical eddy amplitude is $\Delta H \approx H_0$ ($h \approx 1$). The Coriolis force, which is directed toward the center of the eddy (the centrifugal force associated with eddy rotation about its axis can be ignored), prevents a spreading of this anticyclone due to the action of the gravitational and centrifugal force from the global rotation of the liquid. After formation of the local eddy, the disk rotation was stopped and the water surface was photographed with a movie camera, which was located above the container. Since the water was tinted blue, the local mound appeared as a dark spot on the photograph against a white background of the container bottom.

The tests gave the following results.

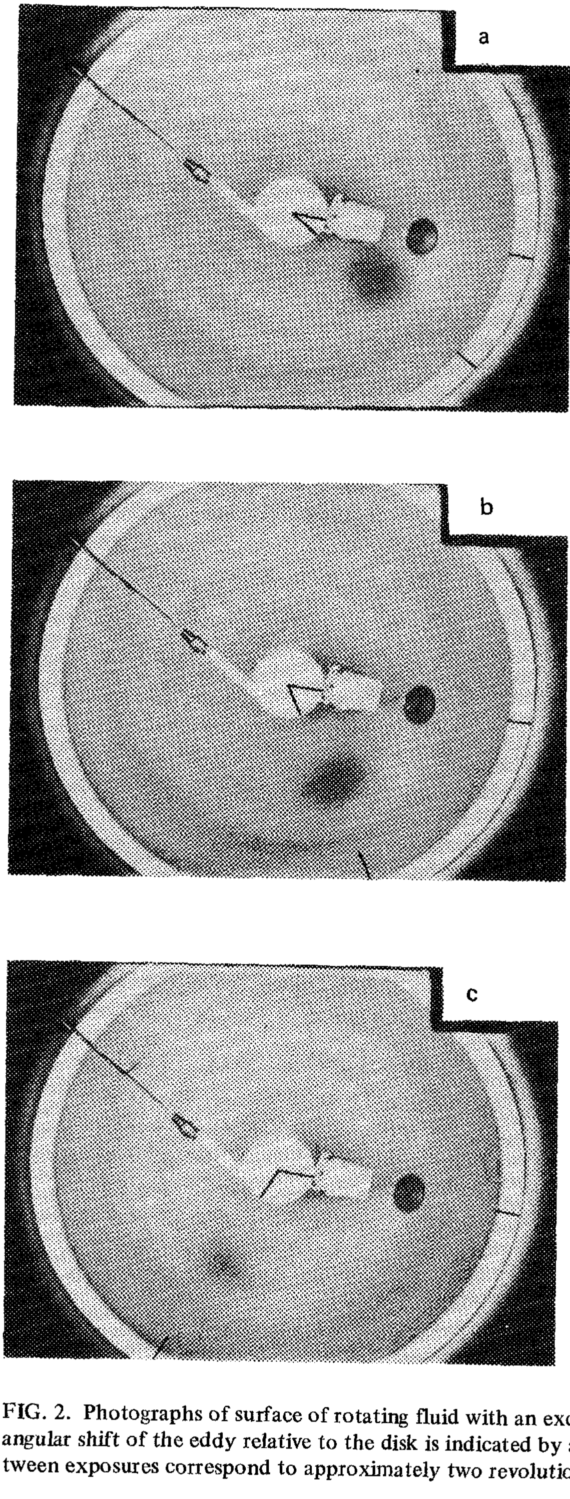

FIG. 2. Photographs of surface of rotating fluid with an excited Rossby soliton (for clarity the angular shift of the eddy relative to the disk is indicated by a dashed line). Time intervals between exposures correspond to approximately two revolutions of the container ($f_0 = 1.8$ Hz).

1′. If the amplitude of the anticyclone above the rotating disk is not too small, then the anticyclonic eddy will break away from the disk immediately after cessation of the "pump" rotation, and the level of water above the disk will drop to that of the surrounding liquid. This corresponds to paragraph 1 of the theoretical conclusions of this paper.

2′. The formed eddy lags behind the globally rotating liquid, i.e., it moves on the paraboloid surface in the opposite direction to its rotation. The lag of the eddy behind the disk producing it amounts to about 15° of arc for each revolution of the paraboloid when $\Delta H = 0.3$ cm (Fig. 2). It is easy to see that this agrees well both qualitatively and quantitatively with paragraph 2 of the theoretical conclusions, since the eddy velocity in the parabolic model must be slightly greater than the phase velocity (4), which amounts to 10° per revolution for $\Delta H = 0.3$ cm.

3′. The characteristic size of the eddy, which remains approximately constant during the observation (Fig. 2), is equal to 5–8 cm, i.e., 3–4 r_0 for $H_0 = 0.3$–0.5 cm and $h \approx 1$. The observed eddy dimensions agree with the theoretical formula (2) (if the substitution $g \to g/\cos \alpha_0$ is taken into account).

The characteristic eddy "lifetime" τ (during which its height decreases to an indistinguishable level) amounts to 8-16 revolutions of the paraboloid, i.e., ~5-10 sec. This time, which is a hundred times longer than that of the gravitational and centrifugal balancing of the excitation, corresponds to the nonlinear character of the studied effect. It agrees approximately with the estimate $\tau \approx (\Delta H)^2/\nu$, where ν is the viscosity of the fluid.

It is important to note that for disk rotation in the "incorrect" direction (coinciding with the direction of container rotation) the Coriolis force is directed from the center of the disk, and a local decrease in the water height is observed instead of a rise. The "depression" formed in this case quickly fills with water after the disk rotation is stopped, and it vanishes during the gravitational and centrifugal balancing.

Thus, the experimental results (1′, 2′, 3′) agree with the theoretical conclusions (1, 2, 3). We note that in comparing experiment with theory we must remember that in the theory the relative amplitude h of the eddy is considered to be a small parameter, whereas in the experiment $h \approx 1$. Nevertheless, it seems to us that our results allow us to draw these conclusions.

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