Is there an upper limit on the pressure in a stellarator?

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According to the present theoretical understanding, no upper limit is set on the pressure in a stellarator by the ideal MHD instability if the system is not too long. The maximum pressure is instead determined by the equilibrium condition. This conclusion refutes an opinion which has been expressed in the literature–that a high value of $\beta = 8\pi p/B^2$ cannot be achieved in a stellarator.

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It is widely believed that one of the disadvantages of the stellarator is a small value of $\beta = 8\pi p/B^2$ (p is the plasma pressure and B is the magnetic field), set by the condition for MHD stability of the plasma.

Let us examine this point. Green *et al.*[1] have derived the following condition for stability with respect to flute perturbations:

$$S_0 = 1 + \frac{2\beta' r}{(r\iota')^2}\,[\iota^2 + N(r^4\iota)'/r^3] > 0, \qquad (1)$$

where ι is the rotational transform, N is the number of periods of the helical winding, and the prime denotes the derivative along the minor radius r. It can be seen from (1) that the plasma is stable only if the system has shear (i.e., only if $\iota' \neq 0$) because of the condition $\beta' < 0$, and the maximum pressure must be kept extremely low.

For example, with $\iota = 0.2 + 0.5(r^2/r_0^2)$, $N = 7$, and $\beta = \beta_0(1 - r^4/r_0^4)$ (r_0 is the radius of the plasma boundary), we have $\max\beta_0 = 0.46\%$.

This conclusion, however, has not been substantiated. Condition (1) was actually derived by ignoring both the finite plasma pressure and toroidal effects. Calculations for a tokamak, on the other hand, have shown that toroidal effects are extremely important, leading to a stabilization of flute perturbations.[2] Corrections for the finite pressure also have a stabilizing effect.[3] It follows from corresponding calculations for a stellarator[4] that both toroidal effects and a finite pressure do, in fact, have a strong stabilizing effect, and if the characteristics of the magnetic confinement system are chosen appropriately, it is possible to remove the upper limit set on the plasma pressure by the MHD instability.

To prove this assertion, we shall use the stability condition which was derived by us in Ref. 4 [see Eq. (35) of that paper], assuming for, simplicity, that the ohmic current is zero and that the helical field is described by a potential which depends on the poloidal azimuthal angle θ and the toroidal azimuthal angle ϕ through the com-

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bination[1] $\theta - N\phi$. We shall also assume that there is an auxiliary, perpendicular, homogeneous field of relative magnitude $\epsilon_\perp = B_\perp/B$. If we use the expression $\xi^* = -\epsilon_\perp R_0/t - 3/R_0 t \int_0^r rt dr$ for the displacement of the vacuum magnetic surfaces, we can write the stability condition as follows:

$$1 - A\beta_o + B_o\beta_o^2 > 0, \tag{2}$$

$$A(r) = -\frac{2f'r}{(rt')^2}\left\{N\frac{(r^4 t)'}{r^3} + 2t^2 - 13 - \frac{\epsilon R_o}{r^3 t^2}(t'r^3)' + 3\left[\frac{(r^3 t')'}{r^3 t^2} \times \int_0^r rt\,dr - \frac{rt'}{t}\right]\right\}, \tag{3}$$

$$B_o = \frac{2f'}{(rt')^2}\frac{(r^3 t')'}{r^2 t^3} R_o^2\, a(r), \quad a(r) = \frac{t}{r^2}\int_0^r r\,dr \int_r^\infty \frac{f'}{t}dr, \quad \beta(r) = \beta_o f(r). \tag{4}$$

It can be seen from (2)-(4) that the condition $t' > 0$, which holds for a classical stellarator, leads to the condition $B_0 > 0$, so that the stability condition becomes more favorable as the pressure is raised. A transverse external field $B_\perp$, on the other hand, may be either stabilizing ($\epsilon_\perp > 0$) or destabilizing ($\epsilon_\perp < 0$). For systems without shear ($t' \equiv 0$), a transverse field does not affect the plasma stability. On the other hand, since the destabilizing (negative) terms become larger, while the stabilizing (positive) terms become smaller with increasing r, the plasma will be stable throughout the volume if condition (2) holds at its boundary, i.e., at $r = r_0$. Finally, condition (2) holds for any β_0 if

$$A(r_o) \leqslant 2\sqrt{B_o(r_o)} \ . \tag{5}$$

For systems without shear ($t' \equiv 0$) this condition becomes

$$4Nt + 2t^2 \leqslant 13 \tag{6}$$

that is, it requires that the aspect ratio R_0/r_0 not be too large. At $t = 0.23$ and $N = 2.5$, for example (these are the values in the W-VIIA stellarator), condition (6) holds with a wide margin.[2]

For systems with a large shear (for definiteness, we assume that $t = r^2/r_0^2\,\Delta t$, $f = 1 - r^2/r_0^2$), condition (5) becomes

$$6N\Delta t + 2(\Delta t)^2 \leqslant 4\frac{R_o}{r_o}(1 + 2\Delta_\perp) + 13 \tag{7}$$

where $\Delta_\perp = \epsilon_\perp R_0/r_0 \Delta t$ is the relative displacement of the magnetic surfaces caused by the transverse field at the plasma boundary.

Since Δt varies nearly in proportion to R_0, if everything else is held constant, condition (7) also imposes a constraint on the major radius of the system. In general, the constraints imposed on N, $t(r)$, and r_0/R_0 depend on the particular functions $f(r)$ and $t(r)$ and can be found without any difficulty. Without going into a more detailed analysis here, we shall simply state that condition (5) is satisfied for all existing stellarators except the Japanese Heliotron E.

Since the condition for stability with respect to ballooning modes[7] is less stringent than condition (2) for stellarators (in contrast to tokamaks[8], it follows from this analysis that the ideal MHD stability does not impose an upper limit on the pressure according to the present theoretical understanding, for systems which are not very long and which satisfy condition (5). The maximum attainable pressure will thus be set by the equilibrium condition,[6,9] and this upper limit may reach a value of $\Delta t/N$ for systems with a high shear, by virtue of condition (5). This value may also be increased slightly by applying a transverse external field to partially cancel the displacement of the magnetic surfaces caused by the magnetic field generated by the plasma. Although such a field ($\epsilon_\perp < 0$) does degrade the stability condition, it may not violate condition (2) if it is applied only at high values of β, so that the stabilizing effect of the pressure overcomes the destabilizing effect of the transverse field. Although the exact maximum value of β is determined by the specific characteristics of the device, a value $\sim 10\%$ seems completely realistic.

[1)] Strictly speaking, this assumption never holds, and there are always satellite harmonics in the expression for the potential at an amplitude determined by the particular configuration of the conductor winding.[5,6] These satellite harmonics must be taken into account in an analysis for any specific system. For example, if the winding is described by $\theta = c \cos \theta + N\phi$, then the satellite harmonics will have a stabilizing effect if $c > 0$ or a destabilizing effect if $c < 0$. If, on the other hand, the winding is approximately an equal-pitch spiral, $\theta = N\phi$, then the satellite harmonics will be relatively unimportant (at least near the axis), and our assumption is justified. Furthermore, this assumption simplifies the analysis, allowing us to avoid the introduction of additional free parameters.

[2)] To determine how well this assertion holds for the W-VIIA device, it would be necessary to carry out a more detailed analysis incorporating the actual magnetic-field configuration (see the preceding footnote).

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