

Temperature dependence of superconducting gap in a strong microwave field

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For a thin superconducting film, which is irradiated by a microwave-frequency field, we have found the value of the energy gap as a function of the irradiation intensity and its frequency within the entire temperature interval.

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Considerable attention has recently been devoted to theoretical and experimental investigation of superconductivity stimulation by an external, high-frequency field.^{1–7} Satisfactory agreement between theory and experiment was achieved in this investigation in the case of weak electromagnetic fields.⁶ In the case of high-intensity fields, however, there is no such agreement, since the available theoretical estimates,^{2,3} which are based on an approximate solution of the self-consistency equation for the gap in a weak field, lead to an unlimited growth of stimulation with increasing pumping amplitude.⁶ We show in this paper that the exact solution of the self-consistency equation in the case of strong fields leads automatically to saturation and subsequent suppression of stimulation by the strong field. We have solved the self-consistency equation in the entire temperature range and for different pump frequencies and intensities and have also analyzed the obtained results from the viewpoint of feasible experiments.

As shown by Eliashberg,¹ the equation for nonequilibrium order parameter Δ has the form

$$1 = \lambda \int \frac{\omega_D}{\Delta (\epsilon^2 - \Delta^2)^{1/2}} d\epsilon, \quad (1)$$

and the nonequilibrium electron-excitation function n_ϵ must be determined from the corresponding kinetic equation (see, for example, Refs. 1 and 3). We must bear in mind that the self-consistency equation, written in the form (1), contains only the kinetic effects in explicit form, which are attributable to the action of the external field on the superconductor. In addition to this influence, the electromagnetic field leads to a direct dynamic suppression of the gap by the time-averaged square of the field; this can be described by an appropriate renormalization of the interaction constant λ . As shown in Ref. 8 in the case of variable rf fields (at $\omega_0^2 > \gamma\Delta$, ω_0 is the frequency of the external field and γ is the energy attenuation of the electron excitations), the dynamic effect is completely analogous to gap suppression by a static magnetic field. Because of this, an appropriate term must be added to the self-consistency equation (1).

It must be recognized, moreover, that at $\omega_0 < 2\Delta$ the total number of quasiparticles cannot be conserved in a superconductor in a variable field. As shown by Schmidt (see Ref. 5), such a "heating" effect is described by an additional term in the self-consistency equation.⁶ As a result, this equation can be written in the form

$$\int_0^{\omega_D} \frac{\tanh[(\xi^2 + \Delta^2)/2T]}{(\xi^2 + \Delta^2)^{1/2}} d\xi - \int_0^{\omega_D} \frac{\tanh(\xi/2T_{s0})}{\xi} d\xi - 2 \int \frac{n_\epsilon^{(1)} d\epsilon}{\Delta(\epsilon^2 - \Delta^2)^{1/2}} - \frac{\pi\alpha}{4T} f_1(T) - 0,11 \frac{\pi}{2} \frac{\alpha}{\gamma_0} \frac{\omega_0^2}{T_{s0}^2} f_2(T) = 0. \quad (2)$$

Here T_{s0} is the junction temperature in the absence of pumping and $n_\epsilon^{(1)} = n_\epsilon - n_\epsilon^{(0)}$ is the nonequilibrium correction to the excitation distribution function. The term proportional to $f_1(T)$ describes the dynamic suppression mentioned above. Using the results of Ref. 9, we can show that $f_1(T)$ has the form

$$f_1(T) = \left(\frac{T}{\Delta} \sinh \frac{\Delta}{T} + 1 \right) / 2 \cosh^2 \frac{\Delta}{2T}. \quad (3)$$

The $f_2(T)$ function in the "heating" term in Eq. (2) can be approximated as $f_2(T) \approx \exp(-\Delta/T)$, if the number of quasiparticles decreases sharply with decreasing temperature.

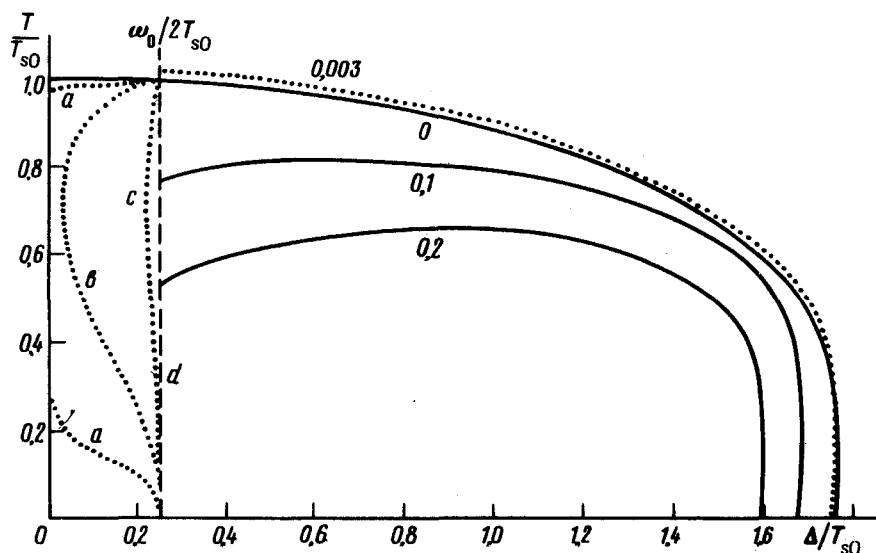


FIG. 1. The $\Delta(T)$ dependence found from the self-consistency equation (2) for $\gamma_0 = 0.01$, $\omega_0 = 0.5$, and the dependence for different α values (the numbers are given on the curves). In the region $\Delta < \omega_0/2$ the parameters for the curves are as follows: (a) $\alpha = 10^{-5}$, (b) $\alpha = 10^{-4}$, (c) $\alpha = 10^{-3}$, (d) $\alpha > 10^{-3}$. (All parameters are given in units of T_{s0} .) The curve for $\alpha = 0$ was determined from the BCS theory.

The expression for $n^{(1)}$ in high fields was found in Refs. 3 and 8 at $T \approx T_s$. This expression, which can be easily generalized to arbitrary temperatures, has the form

$$n^{(1)}(\gamma) = - \frac{\Delta}{4T} \left\{ e^{B\gamma} \int_{\gamma}^{\infty} dx e^{-Bx} \phi(x) + e^{-B\gamma} \left[\int_0^{\infty} dx e^{-Bx} \phi(x) - \int_0^{\gamma} dx e^{Bx} \phi(x) \right] \right\}, \quad (4)$$

$$\phi(\gamma) = - \frac{4\gamma}{(\gamma^2 + 2)(\gamma^2 + 1)^{1/2}} \frac{E(\gamma)}{[1 + E(\gamma)]^2}, \quad E(\gamma) = \exp \left[\frac{\Delta}{T} (\gamma^2 + 1)^{1/2} \right]$$

$$\gamma = (\epsilon^2 - \Delta^2)^{1/2} / \Delta, \quad B = (2\beta)^{1/2}, \quad \beta = \gamma \Delta^2 / 4 \alpha \omega_0^2, \quad \alpha = (e/c)^2 A_{\omega}^2 D,$$

$$D = v_F l / 3.$$

[We further assume that the attenuation γ is caused by electron-phonon processes and use the approximation $\gamma = \gamma_0 (T/T_{s0})^3$.] Note that Eqs. (1)-(4) are generally valid when the conditions

$$\alpha \ll \Delta, \quad \omega_0 \gg \gamma, \quad \tau_{imp} \omega_0 \ll 1, \quad \beta \ll 1 \quad (5)$$

are satisfied.

We solved Eqs. (2)-(4) numerically. The results in Figs. 1 and 2 show that at low temperatures, when the number of quasiparticles is small, the superconductivity is suppressed primarily by the dynamic steaming effect of the electromagnetic field; in this case Δ decreases proportionally to the field amplitude. Nonlinear kinetic effects are important at higher temperatures; specifically, there is a stimulation of superconductivity,⁴ which increases initially with increasing pumping intensity and then becomes saturated¹⁾. At large pump amplitudes the superconductivity is suppressed at all temperatures (Fig. 1). Figure 2 shows the dependence of the maximum stimulation temperature $T_{s \max}$ on the pump amplitude and frequency. It can be seen that an increase of the frequency facilitates stimulation. The boundary between the superconducting state and the normal state for the two pump frequencies $\omega_0 = 0.5$ and $\omega_0 = 0.1$ is shown in Fig. 3 in terms of T versus α . Thus, our calculation

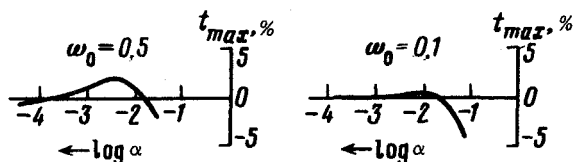


FIG. 2. The quantity $t_{\max} = (T_{s \max} - T_{s0}) / T_{s0}$, is plotted as a function of the pump intensity for different values of ω_0 / T_{s0} (the numbers are given on the curves); $\gamma_0 = 0.01$.

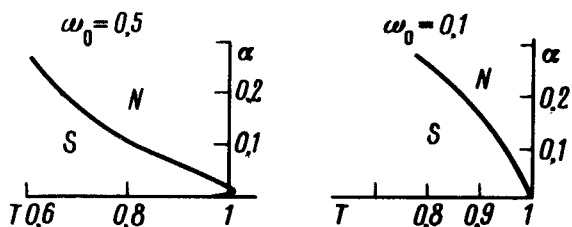


FIG. 3. Boundary between S and N states for $\omega_0 = 0.5$ and $\omega_0 = 0.1$.

scheme, unlike that of Ref. 2, gives at least a qualitatively correct picture at any field intensities.

In the range of values $2\Delta < \omega_0$ the processes of direct production of quasiparticles from the condensate by the electromagnetic field must be taken into account. To do this, we must add a term in Eq. (2) in a linear field approximation,

$$-\frac{4a}{\gamma} \frac{\omega_0}{\Delta} V(\epsilon) \left(1 - n_{\epsilon}^{(o)} - n_{\omega_0 - \epsilon}^{(o)} \right) \frac{d\epsilon}{\epsilon}, \quad V(\epsilon) = \frac{[\epsilon(\omega_0 - \epsilon) - \Delta^2] \theta(\omega_0 - \epsilon - \Delta)}{\{[(\omega_0 - \epsilon)^2 - \Delta^2](\epsilon^2 - \Delta^2)\}^{1/2}}.$$

The order parameter can be drastically reduced in weak fields by taking into account the direct production, and at $\alpha/\gamma_0 \gtrsim 0.1$ the self-consistency equation has no solutions for $\Delta < \omega_0/2$. The appearance of the lower branch of the $\Delta(T)$ solutions (see Fig. 1), which may be associated with hysteresis effects (compare with Ref. 7), is worth noting.

The results of our calculations show that the constraint imposed on the stimulation effect follows directly from theory¹⁻⁴ and does not require the consideration of Joule heating of the samples by a microwave field.⁶ It would be interesting to measure the gap in films in a high-frequency field (at $\omega_0 \sim T_{s0}$) in the entire temperature range.

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¹⁾ A similar comment was made in Ref. 10.

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