The effect of a hot jet in the hydrodynamic collision theory of heavy ions

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The penetration of a nuclear particle through infinite nuclear matter is analyzed. It is shown that a collective jet flow is initiated in it. The temperature of the jet ($T \sim 100$ MeV) is estimated.

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We shall analyze the penetration of a nuclear particle with a velocity $\mathbf{u}(|\mathbf{u}|>s$, where s is the velocity of sound in nuclear matter) through infinite nuclear medium. We shall examine the collective, steady-state motion of the nuclear medium, ignoring the effect of the medium on the particle motion. Such motion of the medium is de-

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scribed by the hydrodynamic equations, which in the coordinate system associated with the incident particle, have the form

$$\operatorname{div} \rho \mathbf{v} = 0,$$

$$\rho (\mathbf{v} \nabla) \mathbf{v} = - \nabla p - \frac{m\mathbf{v}}{\tau} \delta(\mathbf{r}), \tag{1}$$

$$\rho \mathbf{v} \nabla \left(\frac{\mathbf{v}^2}{2} + \frac{5}{2} \frac{p}{\rho} \right) = - \frac{m\mathbf{v}^2}{2\tau} \delta(\mathbf{r}),$$

where ρ, p, and $\mathbf{v}$ is the density, hydrodynamic pressure, and hydrodynamic velocity of the medium, respectively. In contrast to the ordinary hydrodynamic equations, the right-hand side of Eqs. (1) has δ-shaped terms, which describe the effect of the incident particle on the medium. Here τ is the parameter that characterizes the momentum and energy transfer from the incident particle to the medium. Note that such hydrodynamic equations satisfy Galileo's relativity principle. The parameter τ can be calculated on the basis of the Boltzmann kinetic equation in a certain model. Specifically, $\tau = 4 \times 10^{-24}$ sec for a simple, classical, nucleon gas. In writing the system (1) we have also assumed that the medium is described by the ideal gas law, i.e., the expression $w = 5/2\, p/\rho$ is used for enthalpy.

It should be noted that the hydrodynamic equations (1) express the mass, momentum, and energy conservation laws. Moreover, it was assumed in Eqs. (1) that the density tensor of the momentum flow has the form $\Pi_{ik} = \rho v_i v_k + p\delta_{ik}$. This assumption is not a rigorous constraint because the continuous medium, which is in the state of local, thermodynamic equilibrium, is characterized by $v_i(x)$, $\rho(x)$, and $p(x)$ and only one symmetric tensor of second order $v_i v_k$ can be formed from the single v_i vector. This accounts for the form of Π_{ik}. The above facts lead us to believe that Eqs. (1) represent a good model for the description of the collisions of high energy, heavy ions.

We shall examine the case in which the velocity of the incident particle is close to that of sound in the medium. As is well known, a linear approximation can be used in this case, i.e., $\rho = \rho_0 + \rho'$, $p = p_0 + p'$, and $\mathbf{v} = \mathbf{u} + \mathbf{v}'$, where $\mathbf{u}$ is the velocity of incident flow in the system associated with the incident particle. It is directed along the x axis from $+\infty$ to $-\infty$. Thus the linear approximation

$$\rho_0 \operatorname{div} \mathbf{v}' + \mathbf{u} \operatorname{grad} \rho' = 0,$$

$$\rho_0 (\mathbf{u} \nabla) \mathbf{v}' = - \nabla p' - \frac{m\mathbf{u}}{\tau} \delta(\mathbf{r}), \tag{2}$$

$$\rho_0 \mathbf{u} \left\{ (\mathbf{u} \nabla) \mathbf{v}' + \frac{5}{2\rho_0} \nabla p' - \frac{5}{2} \frac{p_0}{\rho_0^2} \nabla \rho' \right\} = - \frac{m\mathbf{u}^2}{\tau} \delta(\mathbf{r}).$$

We shall use the Fourier transform to solve the linear system of differential equations (2). Suppose that $a_\mathbf{k}$, $b_\mathbf{k}$, and $c_\mathbf{k}$ are Fourier transforms of ρ', p', and $\mathbf{v}'$, respectively. They must therefore satisfy the algebraic system of equations whose solution has the form

$$a_{\mathbf{k}} = -i\,\frac{m}{\tau}\left(s^2 + \frac{1}{3}\mathbf{u}^2\right)\frac{(\mathbf{k}\mathbf{u})}{s^2\,((\mathbf{k}\mathbf{u})^2 - \mathbf{k}^2 s^2)} + i\,\frac{m}{3\tau}\,\frac{\mathbf{u}^2}{s^2}\,\frac{1}{(\mathbf{k}\mathbf{u})},$$

$$b_{\mathbf{k}} = -i\,\frac{m}{\tau}\left(s^2 + \frac{1}{3}\mathbf{u}^2\right)\frac{(\mathbf{k}\mathbf{u})}{(\mathbf{k}\mathbf{u})^2 - \mathbf{k}^2 s^2}, \tag{3}$$

$$\mathbf{c}_{\mathbf{k}} = i\,\frac{m}{\tau\rho_0}\left(s^2 + \frac{1}{3}\mathbf{u}^2\right)\frac{\mathbf{k}}{(\mathbf{k}\mathbf{u})^2 - \mathbf{k}^2 s^2} + i\,\frac{m\mathbf{u}}{\tau\rho_0(\mathbf{k}\mathbf{u})}.$$

Using expressions (3) and performing an inverse Fourier transformation, we find

$$p' = \frac{m}{\tau}\left(s^2 + \frac{1}{3}\mathbf{u}^2\right)\frac{|\mathbf{u}|}{2\pi(\mathbf{u}^2 - s^2)}\,G\,\Theta(-x),$$

$$\rho' = \frac{m}{\tau}\left\{\left(s^2 + \frac{1}{3}\mathbf{u}^2\right)\frac{|\mathbf{u}|}{2\pi s^2(\mathbf{u}^2 - s^2)}\,G - \frac{1}{3}\,\frac{|\mathbf{u}|}{s^2}\,\delta^{(2)}(\mathbf{r}_\perp)\right\}\Theta(-x),$$

$$\mathbf{v}' = -\frac{m}{\tau}\left\{\left(s^2 + \frac{1}{3}\mathbf{u}^2\right)\frac{1}{2\pi\rho_0(\mathbf{u}^2 - s^2)}\left(G\mathbf{n}_u + \frac{\sqrt{\mathbf{u}^2 - s^2}}{|\mathbf{u}|}F\mathbf{n}_{r_\perp}\right) + \frac{1}{\rho_0}\,\mathbf{n}_u\,\delta^{(2)}(\mathbf{r}_\perp)\right\}\Theta(-x), \tag{4}$$

where $\mathbf{n}_u = \mathbf{u}/|\mathbf{u}|$, $\mathbf{n}_{r_\perp} = \mathbf{r}_\perp/|\mathbf{r}_\perp|$ are unit vectors $(\mathbf{u}\mathbf{r}_\perp) = 0$, $\Theta(x)$ is a unit function, and F and G functions are determined by the expressions

$$G = \begin{cases} -\dfrac{|x|\,a}{(x^2a^2 - \mathbf{r}_\perp^2)^{3/2}} & ; \\ +\infty & ; \\ 0 & \end{cases} \qquad F = \begin{cases} -\dfrac{|\mathbf{r}_\perp|}{(x^2a^2 - \mathbf{r}_\perp^2)^{3/2}} & \text{for } |x|\,a > r_\perp \\ +\infty, & \text{for } |x|\,a = r_\perp, \\ 0, & \text{for } |x|\,a < r_\perp \end{cases} \tag{5}$$

where

$$a = \frac{s}{\sqrt{\mathbf{u}^2 - s^2}}. \tag{6}$$

The collective motion of nuclear matter described by expressions (4)-(6) has two characteristic features. First, the presence of a Mach cone

$$\sin\beta = s/u, \tag{7}$$

in which the perturbation from a passing particle is propagated. This effect has been discussed in Ref. 1. Second, in contrast to Ref. 1, this collective motion is characterized by "flow separation" (Ref. 2). This effect is distinguished by the fact that Eqs. (4) have δ-shaped terms in ρ' and v', i.e., there is a dislocation line in the medium (a line which extends from $x=0$ to $-\infty$). The dissipative processes must be taken into account in the neighborhood of such special line. Allowance for the dissipative processes leads to smearing out of the δ-shaped terms and to the formation of finite-size regions in which the condition for potential motion assumed in Ref. 1 is violated. Specifically allowance for the viscosity (η is the shear viscosity coefficient) gives the characteristic size of the δ "jet" (Ref. 2)

$$\delta = 2\sqrt{\frac{|x|\eta}{u\rho_0}} \tag{8}$$

and then the δ function is smeared out in the following way

$$\delta^{(2)}(\mathbf{r}_\perp) = \frac{\delta(r_\perp)}{\pi r_\perp} \rightarrow \frac{1}{2\pi r_\perp}\sqrt{\frac{u\rho_0}{|x|\eta}} \cdot \begin{cases} 1, & \text{for } r_\perp \leqslant \delta \\ 0, & \text{for } r_\perp > \delta \end{cases} \tag{9}$$

We can show now that only the jet is heated as a result of such collective excitation of the nuclear medium, and the Mach cone remains at the original temperature. In fact, using the expression for the internal energy of the ideal gas per particle (Ref. 3) $\epsilon=(3/2)(p/\rho)$, we obtain

$$\Delta\epsilon = \frac{3}{2}\frac{p'}{\rho_0} - \frac{3}{2}\frac{p_0}{\rho_0^2}\rho'. \tag{10}$$

On the other hand, $\Delta\epsilon=\Delta\epsilon_x+\Delta\epsilon_T$, where $\Delta\epsilon_x$ is the variation of the internal energy due to compression $\Delta\epsilon_x = p_0/\rho_0^2\,\rho'$, and $\Delta\epsilon_T$ is the variation of the internal energy due to heating $\Delta\epsilon_T=\gamma T^2\rho_0^{-4/3}\epsilon_0\,[\gamma=0.18(m/\hbar^2)^2,\ \epsilon_0=3/5\epsilon_F]$. Substituting all these expressions in (10), we obtain

$$T^2 = \xi\epsilon_0^2\,\frac{p' - s^2\rho'}{s^2\rho_0}, \tag{11}$$

where $\xi\approx 1.15$. Using expressions (4) we can easily see that $p'-s^2\rho'$ is identically equal to zero in the Mach cone and is nonvanishing only in the jet. Therefore, the temperature of the Mach cone is $T=0$ and the temperature of the jet increases and is determined by

$$T^2 = \frac{1}{6\pi}\xi\epsilon_0^2\,\frac{mu}{\tau}\,\frac{1}{s^2\rho_0}\sqrt{\frac{u\rho_0}{|x|\eta}}\,\frac{1}{r_\perp}. \tag{12}$$

If we use the interpolation for the temperature dependence of the viscosity coefficient η(Ref. 4), then we can easily show that the maximum temperature of the jet $T_{max}\approx 2.4$ is $\epsilon_F\approx 100$ MeV. We assumed in these estimates that $\tau=4\times 10^{-24}$ sec. It should be noted that this estimate is clearly overestimated because of the use of the low-temperature expression for the T dependence of $\Delta\epsilon_T$ (at $T<\epsilon_F$).

Thus, as a result of penetration of a nuclear particle through nuclear medium, the latter develops a collective motion in the form of strongly heated ($T \sim 100$ MeV) jet that moves in the direction of motion of the incident particle. This jet apparently leads to an angular distribution of the outgoing particles, which has a maximum in the front hemisphere.

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