Two-dimensional rotons near a helium II-solid boundary

V. N. Bondarev
Scientific-Research Institute of Physics at the I. I. Mechnikov State University, Odessa

(Submitted 1 April 1981)

Piz'ma Zh. Eksp. Teor. Fiz. **33**, No. 10, 511–515 (20 May 1981)

It is shown that a series of localized states corresponding to two-dimensional rotons with smaller characteristic pulses than those of a bulk roton must exist in a superfluid helium near its boundary with the solid substrate.

PACS numbers: 67.40.Db

Recent experiments[1] on inelastic neutron scattering in He-II films in Graphon showed that in addition to a peak $\Delta \approx 0.77$ meV corresponding to the energy of a bulk roton, there is an additional low-energy peak $\Delta_0 \approx 0.54$ meV, whose location is almost independent of the film thickness (the neutron-scattering vector similar to the wave vector corresponding to the roton minimum in He-II was chosen). To determine the nature of the additional peak, Thomlinson *et al.*[1] suggested the existence of special roton excitations in a compressed layer near the helium-solid boundary. We show below that a roton-type quasiparticle in the van der Waals attraction field has a series of surface, discrete levels corresponding to two-dimensional rotons whose dispersion laws, in principle, can be reconstructed from neutron-scattering experiments.

We shall examine a semi-infinite He-II volume bounded by a flat, solid substrate that acts as a source of van der Waals forces, compressing the liquid.[2] The roton "senses" these forces, i.e., the proximity of the helium-solid boundary, because of variation of the roton parameters, primarily the gap $\Delta(\rho)$ which depends on the local helium density ρ.[3] According to the general theory of van der Waals forces,[2] the increment to the density of the liquid at a distance z from the boundary, that is larger than the interatomic distance a, has the form (see, for example, Ref. 4)[1)]

$$\delta\rho(z) = \frac{\hbar}{16\pi^2 u_1^2 z^3}\int_0^\infty d\omega\,[\epsilon(i\omega) - 1] > 0, \tag{1}$$

where u_1 is the velocity of first sound, $\epsilon(i\omega)$ is the dielectric constant of liquid helium at the imaginary frequency, and the bulk density is assumed to be the reference point of the density. In the linear approximation of $\delta\rho$

$$\Delta(z) = \Delta + (\partial\Delta/\partial\rho)_0\,\delta\rho(z) = \Delta - \gamma_3/z^3, \qquad \Delta \equiv \Delta(\infty), \tag{2}$$

and the constant $\gamma_3 > 0$ [for a superfluid helium $(\partial\Delta/\partial\rho)_0 < 0$ (Ref. 5 and 6)]. The component $U(z) = -\gamma_3/z^3 \sim -\Delta/(z/a)^3$ in Eq. (2), which is the potential energy of a roton in the substrate's field, corresponds to attraction.

The steady-state Schrödinger equation for the wave function of a roton Ψ, which belongs to the eigenvalue E,

$$\{\Delta + (\hbar^2\vec{\nabla}^2 + p_0^2)^2/(8\mu p_0^2) + U(z)\}\Psi = E\Psi \tag{3}$$

can be obtained (under similar conditions Lifshitz and Pitaevskiĭ[7] were the first to analyze the quasi-classical scattering of rotons by vortex filaments; see also Ref. 8) by substituting the operator $-i\hbar\vec{\nabla}$ for the momentum $\mathbf{p}$ of a roton in the classical Hamiltonian, which follows from an alternative representation of the spectrum of elementary Landau excitations near the minimum located at $p = p_0$; μ is the effective mass of a roton.

The motion of a roton along the surface—a free motion with the momentum $\mathbf{p}_{\parallel}$—can be separated from the transverse motion. Representing the $\psi(z)$ wave function of the latter in the quasi-classical form[9]

$$\psi(z) = \exp[i\sigma(z)/\hbar], \qquad \sigma(z) = \sigma_0(z) - i\hbar\sigma_1(z) + \ldots, \tag{4}$$

we obtain (the prime denotes the derivative with respect to z)

$$[(\sigma_0')^2 - p_0^2 + p_{\parallel}^2] = 8\mu p_0^2[E - \Delta - U(z)], \tag{5}$$

$$\sigma_1 = -\frac{1}{2}\ln\{\sigma_0'[(\sigma_0')^2 - p_0^2 + p_{\parallel}^2]\}, \ldots \quad .$$

In the simplest case $p_{||}=p_0$ we can see from (5) that the finite motion of a roton corresponds to $E<\Delta$, and the classical reversal points are determined by the equation

$$E-\Delta=U(z_{1,2}). \tag{6}$$

The left reversal point z_1 is determined by the behavior of the potential at a distance of the order of a from the boundary, where the macroscopic approximation breaks down. Because of this, the deep-lying levels of the two-dimensional rotons cannot be analyzed in the context of this approach. As for the weakly coupled states, the classically resolved regions corresponding to them exceed the interatomic distance substantially, and the energies of weakly coupled levels can be determined without the use of a specific model for the He-II–substrate boundary.

The quasi-classical action σ_0 corresponding to the motion of a roton in the classically resolved region must be quantized. On the basis of (5) for $p_{||}=p_0$ we obtain the quantization conditions in the form (this can be confirmed by a detailed analysis)

$$\int_{z_1}^{z_2}\{8\mu p_0^2\,(E-\Delta+\gamma_3/z^3)\}^{1/4}\,dz=\pi\hbar n, \qquad n-\text{integer}, \tag{7}$$

from which we conclude that for weakly coupled states ($n\gg 1$)

$$E_n=\Delta-\frac{(8\mu p_0^2)^3\gamma_3^4 I_0^{12}}{(\pi\hbar n)^{12}}, \quad I_0\equiv\int_0^1 d\xi\left(\frac{1}{\xi^3}-1\right)^{1/4}=\frac{\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{1}{12}\right)}{3\Gamma\left(\frac{4}{3}\right)}\approx 3{,}9. \tag{8}$$

There is, therefore, an infinite number of discrete roton boundary levels at $p_{||}=p_0$; these levels become more crowded at $E=\Delta$.

We shall consider one general result. For values of $p_{||}$ close to p_0 we can assume in Eq. (3) that

$$V(z)=\frac{1}{8\mu p_0^2}\left\{(p_0^2-p_{||}^2)^2+2\hbar^2(p_0^2-p_{||}^2)\frac{d^2}{dz^2}\right\} \tag{9}$$

is the perturbation operator. Proceeding in the usual manner,[9] we determine the modified energy of the nth level with an accuracy to within $(p_0^2-p_{||}^2)$ in first-order perturbation theory

$$E_n(p_{||})=E_n-\frac{p_0^2-p_{||}^2}{4\mu}\;\frac{\hbar^2}{p_0^2}\int\left|\frac{d\psi_n^{(o)}}{dz}\right|^2 dz, \tag{10}$$

where $\psi_n^{(0)}$ is the eigenfunction of Eq. (3) for $p_{||}=p_0$, which belongs to the E_n value. It can be seen from Eq. (10) that the energy of the level decreases with decreasing $p_{||}$. Let us analyze an arbitrary number $n\gg 1$. Since the E_n $(p_{||})$ spectrum cannot end at $E < \Delta$, it is clear that it must pass through a minimum at a certain $p_{\parallel} = p_{n0}$ $< p_0$. We can easily show that the value of p_{n0} is given by the equation

$$p_{n0} \doteq p_0 \left(1 - \frac{\hbar^2}{p_0^2} \int \left| \frac{d\psi_{n0}}{dz} \right|^2 dz \right)^{1/2} < p_0, \tag{11}$$

where ψ_{n0} is the wave function of the nth discrete level, which is the solution of Eq. (3) at $p_{\parallel} = p_{n0}$. The obtained result can be formulated as follows: a reduction in the number of degrees of freedom of a roton reduces (at least for weakly coupled states) the characteristic momentum as compared with the bulk value of p_0. We can assume that this rule applies to the ground state of a two-dimensional roton (the experimental data[1] are consistent with such a assumption).

In conclusion, we note that the low-energy peak found[1] with the help of neutron scattering in the He-II film corresponds to the binding energy of a localized roton $\delta_0 \equiv \Delta - \Delta_0 \approx 2.6$ K. We should like to emphasize again that an interpretation of experiments such as those in Ref. 1 must take into account the excited branches of two-dimensional rotons predicted by us.

It would be beneficial to continue these experiments by means of which an attempt can be made to reconstruct the aforementioned dispersion curves which correspond to different branches of the two-dimensional roton spectrum.

I wish to express my deep gratitude to I. A. Fomin for a useful discussion of the results.

[1)] The case of small values of z compared with the wavelengths $\lambda_0 \sim 1000$ Å characteristic of the absorption spectrum of He is considered.[2] The dielectric constant of the substrate is assumed to be much larger than unity.

1. W. Thomlinson, J. A. Tarvin, and L. Passell, Phys. Rev. Lett. **44**, 266 (1980).
2. I. E. Dzyaloshinskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, Usp. Fiz. Nauk 73, 381 (1961) [Sov. Phys. Uspekhi 4, 153 (1961)].
3. V. N. Bondarev, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 693 (1973) [JETP Letters **18**, 405 (1973)].
4. J. Mahanty and B. W. Ninham, J. Chem. Phys. **59**, 6157 (1973).
5. I. M. Khalatnikov, Teoriya sverkhtekuchesti (Theory of Superfluidity), Nauka, Moscow, 1971.
6. O. W. Dietrich, E. H. Graf, C. H. Huang, and L. Passell, Phys. Rev. **A5**, 1377 (1972).
7. E. M. Lifshitz and L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. **33**, 535 (1957) [Sov. Phys. JETP **6**, 418 (1957)].
8. I. Iguchi, Phys. Rev. **A6**, 1067 (1972).
9. L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Nauka, Moscow, 1974, Quantum Mechanics. 3rd ed. Pergamon Press, Oxford, 1977.

Translated by S. J. Amoretty
Edited by Robert T. Beyer