# Magnetoresistance of thin films and of wires in a longitudinal magnetic field

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(Submitted 2 April 1981)

Pis'ma Zh. Eskp. Teor. Fiz. **33**, No. 10, 515–518 (20 May 1981)

A magnetic field strongly affects the quantum corrections to the conductivity of disordered metal thin films even if the field lies in the plane of the film. A magnetic field also strongly affects the quantum corrections to the conductivity of thin wires. Expressions are derived for the resulting anomalous magnetoresistance.

PACS numbers: 72.15.Gd, 73.60.Dt

Substantial progress has recently been made toward an understanding of the anomalous magnetoresistance. The effect of a magnetic field on the quantum corrections to the conductivity of disordered metals and degenerate semiconductors leads to an anomalous magnetoresistance at magnetic fields which are weak in the classical sense.[1–3] The magnetoresistance of three-dimensional systems and also two-dimensional systems (inversion layers and thin films) was studied in Refs. 1–3 for the case in which the magnetic field was directed perpendicular to the film.

In this letter we study how the resistance of thin films is affected by a magnetic field in the plane of the film; we also study the magnetoresistance of wires.

The basic quantum correction to the conductivity of noninteracting electrons is[4,1]

$$\Delta\sigma = -\frac{2\sigma_0}{\pi\nu} C(\mathbf{r}, \mathbf{r}'). \tag{1}$$

Here $\sigma_0$ is the conductivity of the sample, $\nu$ is the density of state at the Fermi level, and $C(\mathbf{r}, \mathbf{r}')$ is the sum of the fan-shaped ladder diagrams, which satisfies the equation

$$\hbar\left\{D\left[-i\vec{\nabla}_r - \frac{2e}{c}\mathbf{A}(\mathbf{r})\right]^2 + \frac{1}{\tau_\phi}\right\} C(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}'), \tag{2}$$

where $D$ is the electron diffusion coefficient, $\mathbf{A}(\mathbf{r})$ is the magnetic vector potential, and $\tau_\phi$ is the time over which the phase coherence of the electron wave function is disrupted (by inelastic processes, for example).

The boundary conditions on Eq. (2) may be written as follows:

$$\left(\frac{\partial}{\partial \mathbf{n}} + \frac{2ie}{c}\mathbf{A}\mathbf{n}\right) C = 0, \tag{3}$$

where $\mathbf{n}$ is the normal to the surface of the sample.

 0021-3640/81/10499-03$00.60  

In the absence of a magnetic field, the sum $C(\mathbf{r}, \mathbf{r}')$ is

$$C(\mathbf{r}, \mathbf{r}') = \frac{1}{\hbar} \sum_{\mathbf{q}} \left( Dq^2 + \frac{1}{\tau_\phi} \right)^{-1} . \tag{4}$$

If the sample is a film in the $xz$ plane, with a thickness $a$ small in comparison with $L_\phi = \sqrt{D\tau_\phi}$, it is sufficient to retain only the term with $q_y = 0$ in the sum over $q_y$ ($q_y = \pi n/a$, $n = 0, \pm 1, \pm 2, \ldots$). If this film is immersed in a magnetic field $\mathbf{H} \| z$, and if we choose the gauge $A_x = Hy$, $A_y = A_z = 0$, then we can write the solution of Eq. (2) in the form

$$C(y, y') = \int \frac{dq_x dq_z'}{2\pi^2 \hbar} \sum_n \frac{\phi_{n, q_x}(y)\, \phi_{n, q_x}(y')}{Dq_z^2 + E_{n, q_x} + \frac{1}{\tau_\phi}} , \tag{5}$$

where $\phi_{n, q_x}(y)$ are the eigenfunctions, and $E_{n, q_x}$ are the eigenvalues, of the equation

$$-D \left[ \frac{\partial^2}{\partial y^2} - \left( q_x - \frac{2e}{c} Hy \right)^2 \right] \phi_{n, q_x}(y) = E_{n, q_x} \phi_{n, q_x}(y) \tag{6}$$

with the boundary conditions

$$\left. \frac{\partial \phi_{n, q_x}}{\partial y} \right|_{y = \pm a/2} = 0. \tag{6a}$$

In lowest-order perturbation theory with the magnetic field treated as a perturbation, the ground-state energy $E_{0, q_x}$ is

$$E_{0, q_x} = D \left( q_x^2 + \frac{a^2}{12 L_H^4} \right), \tag{7}$$

where $L_H = \sqrt{c\hbar/2eH}$ is the "magnetic length" of a particle with charge $2e$. Substitution of (7) in (5) yields

$$C(\mathbf{r}, \mathbf{r}) = \frac{1}{(2\pi)^2 \hbar D a} \int \frac{dq_x dq_z}{q_x^2 + q_z^2 + a^2/12 L_H^4 + L_\phi^{-2}} . \tag{8}$$

The magnetoconductivity of a square film, therefore, is

$$G(H) - G(0) = \frac{e^2}{2\pi^2 \hbar} \ln \left( \frac{a^2 L_\phi^2}{12 L_H^4} + 1 \right) = \frac{e^2}{2\pi^2 \hbar} \ln \left( \frac{\tau_\phi}{\tau_H} + 1 \right) . \tag{9}$$

The time $\tau_H = 12 L_H^4 / Da^2$ is equal to the characteristic decay time of the correlator of the time-reversal operators from superconductivity theory.[5]

The condition $a \ll L_H$ must hold if perturbation theory is to be applied. If this condition does not hold, the sample behaves as if it were three-dimensional, and the theory derived in Refs. 1-3 can be applied to it.

The magnetic field becomes "strong," i.e., it begins to determine the cutoff of the logarithmic divergence, at $L_H \lesssim \sqrt{aL_\phi}$ or $H \gtrsim c\hbar/2eaL_\phi$. A longitudinal field is much more effective than a field directed perpendicular to the film; in the latter case the field is "strong" under the condition $L_H \lesssim L_\phi$.

The magnetoconductivity of wires with transverse dimensions small in comparison with $L_\phi$ is determined in a corresponding way. The correction to the conductivity per unit length in the presence of a magnetic field for such a wire is

$$\Delta G(H) = \frac{e^2}{\pi^2 \hbar}\left(\frac{1}{D\tau_\phi} + \frac{1}{D\tau_H}\right)^{-1/2}. \tag{10}$$

The time $\tau_H$ depends on the direction of the magnetic field and the shape of the sample. For a wire of circular cross section with a radius $R$, in a field directed parallel to the axis of the wire, this time is $\tau_H = 8L_H^4/DR^2$. For a wire of rectangular cross section in a field parallel to one of the sides of the rectangle (i.e., perpendicular to the wire axis) the time is $\tau_H = 12L_H^4/Da^2$, where $a$ is the dimension of the wire in the perpendicular direction to the field.

A longitudinal magnetic field has analogous effects on the quantum corrections to the conductivity for the Cooper interaction of electrons.[3] The only difference is that $L_\phi$ is replaced everywhere by $L_T = \sqrt{\hbar D/T}$.

The fact that a magnetoresistance was not observed for wires in Ref. 6 may mean that the temperature dependence of the conductivity in those experiments was a consequence of a diffusion interaction of the electrons (i.e., at a small momentum difference).

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Translated by Dave Parsons
Edited by S. J. Amoretty