Phenomenology of a τ lepton with a heavy neutrino in the $SU(5)$ model

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The corollaries of phenomenological theory of a τ lepton in asymptotically free $SU(5)$ model, which confirms the existence of a heavy neutrino ($m_{N_\tau} \approx 1.5$ GeV), are given. The $\tau \to 3e$ and $\tau \to e + 2\mu$ decays are shown to be a good test of this model. The decay modes of an N_τ lepton are discussed briefly.

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The asymptotically free $SU(5)$ model of a single interaction[1] is now a well-developed scheme of the grand unification theory, which must be compared with experiment. This model is unique because it unifies in a minimum attainable way the $SU(2) \times U(1)$ Weinberg-Salam model and the $SU(3)$ quantum chromodynamics, which are now considered by many investigators to be the fragments of a future theory. The internal consistency of this theory allows us to hope that its predictions can be made with sufficiently high degree of certainty, which is extremely important now in the formulation of experimental phenomenology of high energy and superhigh energy physics. Several quite interesting predictions of this theory (for example, nonzero mass of e and μ neutrinos,[2] finite lifetime of a proton, and several others) can today be considered worthy of experimental investigation.

The phenomenology of a τ lepton with a "heavy" neutrino is also considered a very interesting feature of this $SU(5)$ theory. The intrinsic, "heavy" neutrino (N_τ lepton with $m_{N_\tau} \approx 1.5$ GeV) is responsible for only a small number of τ-lepton decay channels, whereas the main channels of its decay involve a light ν neutrino (the four-component Dirac spinor). The latter interacts with e and μ leptons according to the Zel'dovich-Mahmound-Konopinski scheme.[3] The phenomenology of a τ lepton is established by mixing (e, τ) leptons with (ν, N_τ) neutrinos

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$$J_\mu^+ = \frac{g}{\sqrt{2}}\left\{\operatorname{tang}(\widehat{e_L, \tau_L})\left(1 + \left|\frac{m_\tau}{m_{N_\tau}}\right|\right)(\bar{\tau}_L\gamma_\mu\nu_L - \bar{e}_L\gamma_\mu N_{\tau L} + \right.$$

$$\left. + \frac{1 - \operatorname{tang}^2(e_L, \tau_L)\dfrac{m_\tau^2}{m_{N_\tau}^2}}{1 + \dfrac{1}{2}\operatorname{tang}^2(\widehat{e_L, \tau_L})\left(1 + \dfrac{m_\tau^2}{m_{N_\tau}^2}\right)}(\bar{e}_L\gamma_\mu\nu_L + \bar{\tau}_L\gamma_\mu N_{\tau L}) + \ldots\right\}$$

$$= \frac{g}{\sqrt{2}}[0{,}44(\bar{\tau}_i\gamma_\mu\nu_L - \bar{e}_L\gamma_\mu N_{\tau L}) + 0{,}9(\bar{e}_L\gamma_\mu\nu_L + \bar{\tau}_L\gamma_\mu N_{\tau L}) + \ldots]. \tag{1}$$

The mixing of (μ, ϑ) leptons [as well as (b, d) quarks] with (ν_R, N_ϑ) neutrinos must be small to avoid changing the ratio of weak constants, that is realized by the μ and β decays and that has been determined experimentally with great accuracy

$$[\sigma(\beta)/\sigma(\mu)] = 1{,}003 \pm 0{,}004. \tag{2}$$

The neutral lepton current causes a number of so-called exotic, τ-lepton decays via (e_R, τ_R) mixing,

$$J_\mu = \frac{g}{\cos\theta_W}\left[\left(\frac{1}{2} - \sin^2\theta_W\right)(\bar{e}_L\gamma_\mu e \quad \bar{\mu}_R^+\gamma_\mu\mu_R^+) - \frac{1}{2}\operatorname{tang}(\widehat{e_R, \tau_R})\right.$$

$$\times (\bar{e}_R\gamma_\mu\tau_R + \ldots)$$

$$\left. - \sin^2\theta_W(\bar{e}_R\gamma_\mu e_R - \bar{\mu}_L^+\gamma_\mu\mu_L^+) + \ldots\right] = \frac{g}{\cos\theta_W}[0{,}29(e_L\gamma_\mu e_L$$

$$- \bar{\mu}_R^+\gamma_\mu\mu_R^+) - 2{,}2\cdot 10^{-2}(\bar{e}_R\gamma_\mu\tau_R + \ldots) - \bar{\nu}\gamma_\mu\gamma_5\nu + \ldots]; \tag{3}$$

where $\tan(\widehat{e_R, \tau_R}) = 4.4\times 10^{-2}$. This $SU(5)$ scheme differs in this respect from the other theories in which these decays are forbidden. Here $\sin^2\theta_W = 0.21$.

Our goal in this letter is to show that currently available experimental data for the phenomenology of a τ lepton are consistent with the scheme is which the "intrinsic," light neutrino in a τ lepton is missing. Such a scheme, which does not violate the standard τ phenomenology, predicts quite interesting effects, specifically, the existence of so-called exotic processes [$\tau\to 3e$ ($e2\mu$) decays and other decays] which have not been observed until now.

We shall use the $\tau\to\nu + X$ decays to describe the phenomenology of a τ lepton. The main τ-lepton decays proceed in this scheme via the $\tau\to e + 2\nu$ and $\tau\to\mu + 2\nu$ channels. The $\tau\to e + 2\nu$ decay is completely analogous to the μ decay, in which a correction is introduced only for the m_μ/m_τ mass ratio. We shall take this important feature into account below. The standard calculations of the $\tau\to e+2\nu$ decay width, which are omitted here, give the following result:

$$\Gamma_{\tau\to e+2\nu}=\frac{2(a^2+2\beta^2)\,m_\tau^5}{192\,\pi^3}\quad, \tag{4}$$

where the α and β coefficients

$$a=\frac{0{,}9\times 0{,}44\,g^2}{2\times 4\times m_W^2};\quad \beta=\frac{2{,}2\times 10^{-2}\,g^2}{2\times 4\,(m_z\cos\theta_W)^2}; \tag{5}$$

which were calculated by using Eqs. (1) and (3), determine the relative contribution of the charged and neutral currents to this decay. The lifetime of a τ lepton can be determined by comparing Eq. (4) with the $\mu\to e+2\nu$ decay width

$$\Gamma_{\mu\to e+2\nu}=\frac{2a_1^2}{192\,\pi^3}\,m_\mu^5,\quad a_\lambda=\frac{0{,}99\times 0{,}9\,g^2}{2\times 4\times m_W^2}\quad, \tag{6}$$

and also by using the experimental values for the ratio of the partial widths[4]

$$b_e=\frac{\Gamma(\tau\to e+2\nu)}{\Gamma(\tau\to \text{all})}=0{,}17 \tag{7}$$

and the lifetime of μ mesons ($T_\mu=2.2\times 10^{-6}$ sec). Finally, we can see that the lifetime of a τ lepton

$$T_\tau=\left(\frac{a_1}{a}\right)^2\frac{1}{1+2\,(\beta/a)^2}\left(\frac{m_\mu}{m_\tau}\right)^5 T_\mu B_e=0{,}98\times 10^{-12}\ \text{sec}, \tag{8}$$

just as in Eq. (8), is very close to its experimental limit [$T_\tau\leqslant 2.3\times 10^{-12}$ sec (Ref. 4)], although it differs from the predictions of the standard $SU(5)$ model with a "light" neutrino, where $T_\tau=2.8\times 10^{-13}$ sec. We do not view this problem, however, as a deficiency of the $SU(5)$ model which is considered here. The ratio of partial widths of the decay of a τ lepton into e and μ leptons, which was calculated according to Eqs. (1) and (3), has also been confirmed experimentally:

$$\frac{\Gamma(\tau\to\mu+2\nu)}{\Gamma(\tau\to e+2\nu)}\approx 1{,}2\ . \tag{9}$$

The agreement with the experimental value [1.06 ± 0.1 (Ref. 4)] is within the error limits.[4] On the whole, we found that the τ-lepton decay according to the scheme $\tau\to\nu+X$ is in sufficiently good agreement with the experiment.

We shall consider those exotic τ-lepton decay channels which set this $SU(5)$ theory apart from the other models. The $\tau\to 3e$ and $\tau\to e+2\mu$ decays are the main decays here. The calculations were carried out according to Eq. (3), taking into account that the particles in the final state are identical:

$$\Gamma_{\tau\to 3e}=\frac{2\,(2\,a_2^2+4\,\beta_2^2)\,m_\tau^5}{192\,\pi^3};\quad \Gamma_{\tau\to e+2\mu}=\frac{2(a_2^2+\beta_3^2)\,m_\tau^5}{192\,\pi^3}\,. \tag{10}$$

The α_i and β_i coefficients are

$$\alpha_2 = \frac{1{,}2\times 10^{-2}\cdot 0{,}29\ g^2}{2\times 4\ (m_z \cos\theta_W)^2}\ ;\ \beta_2 = \frac{1{,}2\times 10^{-2}\times 0{,}21\ g^2}{2\times 4\times (m_z \cos\theta_W)^2};\ \beta_3 = \frac{1{,}2\times 10^{-2}\times 0{,}14\ g^2}{2\times 4\times (m_z \cos\theta_W)^2}. \tag{11}$$

In deriving (11) we have used the expression for the neutral current of the (μ, ϑ) series of particles in addition to (3)

$$J_\mu = \frac{g}{\cos\theta_W}\left[\left(-\frac{1}{2} + \sin^2\theta_W\right)\overline{\mu_R^+}\,\gamma_\mu\,\mu_R^+ + \left(\sin^2\theta_W - \right.\right.$$

$$\left.\left.-\frac{1}{2}\sin^2(\widehat{\mu_L, \vartheta_L})\right)\overline{\mu_L^+}\,\gamma_\mu\mu_L^+ + \ldots\right] = \frac{g}{\cos\theta_W}[-0{,}29\,\overline{\mu_R^+}\,\gamma_\mu\mu_R^+ - 0{,}14\,\overline{\mu_L^+}\,\gamma_\mu\mu_L^+ + \ldots], \tag{12}$$

where it is assumed that $\sin^2(\mu_L, \vartheta_L)\times \approx 0.14$.[1] The relative contribution of $\tau\to 3e$ and $\tau\to e+2\mu$ processes to the total τ-decay probability can be determined by comparing Eq. (10) with Eq. (4) and by using the experimental result of (7)

$$\frac{\Gamma(\tau\to 3e)}{\Gamma(\tau\to \text{all})} \approx 2\times 10^{-3}\% \qquad \frac{\Gamma(\tau\to e+2\mu)}{\Gamma(\tau\to\text{all})} \approx 1\times 10^{-3}\%. \tag{13}$$

Although the estimates of (13) are below the level of experimental constraints,[4] a search for these processes is very important.

The τ-lepton decays, which are accompanied by an intrinsic heavy neutrino, are also very important. Because of the instability of N_τ neutrinos such a decay causes a cascade decay

$$\begin{array}{l} \tau \to N_\tau + e + \nu \\ \quad\ \ \llcorner\!\!\longrightarrow 2e + \nu(e+\mu+\nu) \end{array} \tag{14}$$

which produces three light leptons and two final-state neutrinos. The probability of $\tau\to N_\tau + e + \nu$ decay can be determined from Eq. (1), assuming that the energy release in the final state is small

$$\Gamma_{\tau\to N_\tau + e + \nu} = \frac{2(\tilde{\alpha}^2 + \tilde{\beta}^2)}{8\pi^3}(m_\tau - m_{N_\tau})^5;$$

$$\tilde{\alpha} = \frac{0{,}9\times 0{,}9\,g^2}{2\times 4\times m_W^2};\quad \tilde{\beta} = \frac{0{,}9\times 0{,}98\ g^2}{2\times 4\times m_W^2}. \tag{15}$$

The relative probability of this decay channel can be determined by comparing Eq. (15) with Eq. (4)

$$\frac{\Gamma(\tau \to N_\tau + e + \nu)}{\Gamma(\tau \to \text{all})} = 24\, \frac{\tilde{a}^2 + \tilde{\beta}^2}{a^2 + \beta^2} \left(\frac{m_\tau - m_{N_\tau}}{m_\tau}\right)^5 (17\%) \approx 0{,}1\%, \tag{16}$$

where it is assumed[1] that $m_{N_\tau} \approx 1.46$ GeV. An analogous decay process $\tau \to N_\tau + \mu + \nu$ is suppressed, since the energy release in this case is smaller because of the finite mass of a μ meson.

We shall also estimate the decay probability of an N_τ lepton. Its main decay channels $(2e+\nu)$ and $(e+\mu+\nu)$ in the $SU(5)$ model[1] have approximately the same probability which is calculated here from Eq. (1)

$$\Gamma_{N_\tau \to 2e+\nu} = \frac{2(a'^2 + \beta'^2)}{192\,\pi^2} m_{N_\tau}^5 \;;$$

$$a' = \frac{0{,}44 \times 0{,}9\, g^2}{2 \times 4 \times m_W^2} \;; \quad \beta' = \frac{4{,}4 \times 10^{-2} \times 0{,}9\, g^2}{2 \times 4 \times m_W^2}\,. \tag{17}$$

We can see in Eq. (17) that there is an analogy between the decay of a τ lepton via the $(e+2\nu)$ channel and the corresponding decay of a N_τ neutrino. An analysis of other N_τ-lepton decay channels shows that this analogy applies to the N_τ-lepton phenomenology as a whole, i.e., the relative probabilities of the corresponding decays are similar

$$\Gamma(N_\tau \to 2e + \nu) / \Gamma(N_\tau \to \text{all}) \approx 0{,}17\,. \tag{18}$$

This makes it possible to estimate the lifetime of an N_τ lepton in a very simple way

$$T_{N_\tau} \approx \frac{\Gamma(\tau \to e + 2\nu)}{\Gamma(N_\tau \to 2e + \nu)} \, \frac{1}{\Gamma(\tau \to \text{all})} \approx \left(\frac{m_\tau}{m_{N_\tau}}\right)^5 T_\tau \approx 2{,}8 \cdot 10^{-12}\ \text{sec} \tag{19}$$

by effectively using only the appropriate mass factor.

The obtained results, therefore, confirm the existence of a heavy neutrino that accompanies a τ lepton. The $\tau \to 3e$, $\tau \to e + 2\mu$ and other processes predicted by this model may serve as a good test for distinguishing experimentally this model from that with a light neutrino. It is important to realize, however, that the parameters of this model, when necessary, can be changed to some extent and that the estimates obtained here should be further refined. The qualitative picture of this phenomenology, however, must certainly be preserved.

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