Baryon considered as a soliton in loop space

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The baryon mass for large N is expressed in QCD in terms of the collective field in loop space, which satisfies the nonlinear functional-integral equation. This collective loop field is a relativistic generalization of the self-consistent Witten field. Our approach confirms Witten's idea that a baryon is a soliton in $1/N$ expansion.

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In a recent paper Witten[1] analyzed qualitatively the properties of baryons in QCD for large N. He mentioned that the quark density in a baryon increases proportionally to N for a coupling constant $g_0^2 = \lambda_0/N$. This leads to a self-consistent field theory in a nonrelativistic system. The baryon mass in this picture is equal to N times the average quark energy in a self-consistent field

$$m_B = N < \epsilon > . \tag{1}$$

This resembles solitons in standard field theories in which the mass is also proportional to the inverse coupling constant whose coefficient is expressed directly in terms of the classical field.

Witten's analysis, however, is purely nonrelativistic. Our goal is to formulate a relativistic approach for the description of polychromatic baryons. We shall introduce as an analog of the self-consistent field the amplitude of propagation of the quark in question in the baryon along the world line Γ_{xy} (beginning at the point x and ending at the point y). As a functional of the Γ_{xy} path this amplitude satisfies certain classical nonlinear equations, as will be shown below. There is evidence that these equations can be solved in terms of the fermion string.[2]

Let us solve the equations. A simple local field with baryon quantum numbers has the form

$$B(x) = \frac{1}{\sqrt{N!}} e_{c_1 \cdots c_N} (\bar{\lambda}_{s_1 f_1} q_{c_1}^{s_1 f_1}) \cdots \left(\bar{\lambda}_{s_N f_N} q_{c_N}^{s_N f_N}\right), \tag{2}$$

where q_c^{sf} is the quantum field with the color index c, spin index s and flavor index f, and λ_{sf} is a certain constant spinor in (flavor x spin) space. To find the operator with a certain value of spin and flavor, we must rotate $\lambda^{sf} \rightarrow \Omega_{s'}^{s} \Omega_{f'}^{f} \lambda^{s'f'}$ and average it over all rotations with appropriate weight. This procedure will be described in detail in a study in collaboration with Kostov.

A two-point Green's function

$$G(x - y) = < \bar{B}(x) B(y) >_{\text{QCD}} \tag{3}$$

after elimination of quark fields, reduces to

$$G(x-y) = \frac{\langle \det S_{xy}(A) \exp(\mathrm{Tr}\ln(\hat{\nabla}(A)+m))\rangle_A}{\langle \exp(\mathrm{Tr}\ln(\hat{\nabla}(A)+m)\rangle_A} \quad . \tag{4}$$

Here $\hat{\nabla}(A)$ is a covariant Dirac operator in external gauge field A_μ and

$$S_{xy}(A) = \bar{\lambda} \langle x | (\hat{\nabla}(A) + m)^{-1} | y \rangle \lambda \tag{5}$$

is the corresponding Green's function. The track in functional space is denoted by Tr in the exponential function in Eq. (4).

The Green's function (4) can be represented as a path integral over the paths in phase space

$$S_{xy}(A) = \sum_{\Gamma_{xy}} I(\Gamma_{xy})\, U(\Gamma_{xy}, A), \tag{6}$$

where

$$\sum_{\Gamma_{xy}} \ldots = \int_0^\infty ds \int_{x(0)=x}^{x(s)=y} D\,x(t)\ldots, \tag{7}$$

$$I(\Gamma_{xy}) = \bar{\lambda} \int Dp(t)\, \hat{T} \exp\left[\int_0^s dt\, (p_\mu(x_\mu - y_\mu) - m)\right]\lambda, \tag{8}$$

$$U(\Gamma) = P \exp\left(\int_\Gamma A_\mu\, dx_\mu\right). \tag{9}$$

We can now introduce the inclusive propagation amplitude of the given quark along the fixed world line

$$b(\Gamma_{xy}) = (1/N)\frac{\delta \ln G}{\delta I(\Gamma_{xy})} = (1/N)\frac{\langle \mathrm{tr}[U(\Gamma_{xy}) S_{xy}^{-1}]\det S_{xy} \exp \mathrm{Tr} \ln(\hat{\nabla} + m)\rangle_A}{\langle \det S_{xy} \exp \mathrm{Tr} \ln(\hat{\nabla} + m)\rangle_A}. \tag{10}$$

Because of uniformity in $I(\Gamma)$, the amplitude satisfies the normalization relation

$$\sum_{\Gamma_{xy}} I(\Gamma_{xy})\, b(\Gamma_{xy}) = 1. \tag{11}$$

For large N we can use the factorization of gauge invariant operators and replace Eq. (10) by

$$b(\Gamma_{xy}) \underset{N=\infty}{=} \left\langle \frac{\mathrm{tr}}{N} [U(\Gamma_{xy}) S_{xy}^{-1}] \right\rangle_A . \tag{12}$$

We now can use for (12) the loop operator

$$L_\nu(z) = (1/N) g_0^2\, \partial_\mu\, \delta / \delta\sigma_{\mu\nu}(z). \tag{13}$$

Using a procedure analogous to that in Ref. 3, we find

$$L_\nu(z)\, b\,(\Gamma_{xy}) = < \frac{\mathrm{tr}}{N} (\nabla_\mu F_{\mu\nu}(z)\, U(\Gamma_{zy})\, S^{-1}_{xy}\; U(\Gamma_{xz}) > \;=$$

$$= < \frac{\mathrm{tr}}{N} \left(\frac{\delta}{\delta A_\nu(z)} U(\Gamma_{zy}) S^{-1}_{xy}\; U(\Gamma_{xz}) \right) >. \qquad (14)$$

Calculating the variation in A_ν analogously to that in Ref. 3 and using the factorization, we obtain the sought-for equation

$$L_\nu(z)\, b\,(\Gamma_{xy}) = \int_{\Gamma_{xy}} dt_\nu \delta^{(d)}(t-z)\,[m(\Gamma_{zt})\, b\,(\Gamma_{xt}\Gamma_{zy})\;\theta(t,z) +$$

$$+ m(\Gamma_{tz})\, b\,(\Gamma_{xz}\Gamma_{ty})\,\theta(z,t)\,] - \sum_{\Gamma'_{xy}} I(\Gamma'_{xy}) \int_{\Gamma'_{xy}} dt_\nu \delta^{(d)}(t-z)\, b\,(\Gamma_{xz}\Gamma'_{ty}) \times$$

$$\times\, b\,(\Gamma'_{xt}\Gamma_{zy}). \qquad (15)$$

Here $\theta(z, t)$ is a step function which orders the points z and t along the Γ_{xy} path, and

$$m(C) = (1/N) < \mathrm{tr}\, U(C) > \qquad (16)$$

is the standard Wilson's average which satisfies the equation

$$L_\nu(x)\, m(C) = \int_{C_{xx}} dy_\nu\, \delta^{(d)}(x-y)\, m(C_{xy})\; m(C_{yx}). \qquad (17)$$

Using the methods developed in Ref. 3, we can formulate a perturbation theory for the quantity $b(\Gamma_{xy})$, where the b functional plays the role of the dynamic part of the propagation amplitude of an individual quark in a baryon. The first and second terms in (15) arise when the world line Γ_{xy} describes a closed loop Γ_{zt} (or Γ_{tz}). The third term in (15) describes the exchange interaction of the given quark with the remaining quarks in the baryon. In contrast to the first two terms, this term survives even in the nonrelativistic limit.

As regards the relation (1) for baryon mass, it is preserved in our case

$$m_B = N \int d\,\Omega\; p_0\;, \qquad (18)$$

where the averaging is carried out with the weight

$$\int d\Omega\ldots = \bar{\lambda} \int_0^\infty ds \int_{x(0)=(0,T)}^{x(s)=0} Dx(t)\, Dp(t)\; \hat{T} \exp\left(\int_0^s dt\, (p_\mu(\dot{x}_\mu - \gamma_\mu) - m)\right) \lambda\, b\,(\Gamma_{0T}) \ldots . \qquad (19)$$

To obtain (18), we must use the asymptotic relation

$$m_B = - \lim_{T\to\infty} \frac{\partial \ln G(T)}{\partial T} \qquad (20)$$

and the chain rule of differentiation

$$\frac{\partial \ln G}{\partial T} = \sum_{\Gamma_{0T}} \frac{\delta \ln G}{\delta I(\Gamma_{0T})} \frac{\partial I(\Gamma_{0T})}{\partial T} . \qquad (21)$$

The derivative $\partial I(\Gamma_{OT})/\partial T$ gives the time-dependent component p_0 of the momentum under the integral over the paths. The factor in front of $\partial I/\partial T$ reduces to $b(\Gamma)$ by definition.

We note that the relation (18) is purely classical, consistent with Witten's reasoning.[1]

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