Allowance for the correlation of the center of mass of the target nucleus in the calculations of the cross sections for inclusive reactions

Z. Omboo, A. S. Pak, S. B. Saakyan, A. V. Tarasov, and V. V. Uzhinskiĭ
Joint Institute for Nuclear Research

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The problem of correlations of the center of mass of the target nucleus in the calculations of cross sections for inclusive reactions of hadron-nuclear and nuclear-nuclear interactions is solved.

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The calculation of the cross sections of hadron-nuclear and nuclear-nuclear interactions can be simplified appreciably in the context of multiple-scattering theory (MST)[1] if the distribution of nucleons in the nuclei is assumed to be completely uncorrelated. In analyzing the processes with light nuclei, the most important correlation effects of all the existing effects are governed by the so-called correlations of the center of mass of the nucleus, which are linked to a constraint of the type $\Sigma_{i=1}^{A}\mathbf{r}_i=0$, imposed on the integration range of the coordinates $\mathbf{r}_i$ of the nucleons of a nucleus A in the calculation of the amplitudes of the process in question. Relatively simple "decorrelation" rules, which link the cross sections that were calculated by allowing for the center-of-mass correlations, with those that were calculated without regard for them, can be formulated in the nuclear shell model with oscillator wave functions for the calculation of certain cross sections of hadron-nuclear interactions. Thus, for example, the "correlated" cross sections are linked with the "uncorrelated" cross sections of elastic hadron-nuclear scattering in the following way[2]:

$$\left(\frac{d\sigma_{\text{elast.}}}{dt}\right)_{\text{cor.}} = K^2(t)\left(\frac{d\sigma_{\text{elast.}}}{dt}\right)_{\text{uncor.}}, \qquad K(t) = e^{-t/4A\alpha}. \tag{1}$$

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where A is the number of nucleons in the nucleus and α is its oscillator parameter. However, the cross sections for scattering of a particle by a nucleus, which were summed over every kind of excitation of the target using the completeness condition $\Sigma_f |f\rangle\langle f| = 1$, are "invariant" with respect to the center-of-mass correlations[3]

$$\left(\sum_f \frac{d\sigma_{if}}{dt} \right)_{\text{cor.}} = \left(\sum_f \frac{d\sigma_{if}}{dt} \right)_{\text{uncor.}} ; \qquad \frac{d\sigma_{ii}}{dt} = \frac{d\sigma_{\text{elast.}}}{dt} . \tag{2}$$

We obtain from Eqs. (1) and (2)

$$\left(\sum_f \frac{d\sigma_{if}}{dt} \right)_{\text{cor.}} = \left(\sum_{f \neq i} \frac{d\sigma_{if}}{dt} \right)_{\text{uncor.}} + \left(1 - K^2(t) \right) \left(\frac{d\sigma_{\text{elast.}}}{dt} \right)_{\text{uncor.}} \tag{3}$$

It follows from this that the "decorrelation" rules for cross sections $d\sigma_{if}/dt$ of excitation of isolated levels or for a nuclear disintegration, and hence for the momentum spectra of a scattered particle, which are determined by

$$\frac{d\sigma}{dt\,dp} = \frac{dE}{dp} \sum_f \frac{d\sigma_{if}}{dt} \delta(E - E_o + \epsilon_f - \epsilon_i),$$

where E_0 and E are the energies of a particle before and after scattering, and ϵ_i and ϵ_f are the energies of the original and final states of the target, are generally nontrivial. As emphasized by Azhgirey *et al.*,[4] the investigation of momentum spectra of particles or of light nuclei scattered by light nuclear targets, in which a sufficiently large momentum is transferred to the latter ($q \geqslant 1$ GeV/c), is of particular interest. Since the probability of a total disintegration of a light nucleus in this case is evidently close to unity, we can select with a good accuracy a system of plane waves, which describe the (quasi) free motion of the fragments (nucleons) of a nucleus, as a complete system of wave functions of the final state of the target nucleus in a theoretical analysis of such processes in the context of the MST.

If the Gaussian parametrization

$$|\Phi_o(\mathbf{r}_1, \ldots, \mathbf{r}_A)|^2 \sim \left(\prod_{i=1}^{A} \exp(-\alpha r_i^2) \right) \delta\left(\sum_{i=1}^{A} \mathbf{r}_i \right)$$

is selected, as usual, for the wave function of the ground state of a nucleus A, then, using the standard method of "symmetric" elimination of the δ function (Gartenhaus-Schwartz transformation),[2,3] we can establish a simple correlation between the characteristic functions

$$\Phi(a, t) = \int \frac{d\sigma}{dt\,dE} \exp\left[2iam_N(E - E_o)\right] dE \tag{4}$$

of the process under consideration, which were calculated allowing for the correlations of the center of mass of the nucleus A and without them. These characteristic functions have the following form:

$$\Phi(a, t)_{\text{cor.}} = \exp\left(- \frac{ita}{(1 + i\alpha a)A} \right) \Phi(a, t)_{\text{uncor.}} . \tag{5}$$

Since

$$\Phi(0,\ t) = \sum_f \frac{d\sigma_{if}}{dt}$$

by definition (4), the relation (5) must be consistent with the result (2) obtained in Ref. 3.

The coupling between the momentum spectra, more precisely, between the energy spectra of particles scattered by "correlated" and "uncorrelated" targets, which follows from Ref. 5, is an integral coupling

$$\left(\frac{d\sigma}{dt\,dE}\right)_{\text{cor.}} = e^{-\frac{t}{A\alpha}} \left\{ \left(\frac{d\sigma}{dt\,dE}\right)_{\text{uncor.}} - \frac{2M_A}{A\alpha} \int_E^{E_M} dE' \sqrt{\frac{t}{2M_A(E-E')^*}} \right\}, \tag{6}$$

$$I_1\left(\frac{2\sqrt{2M_A t(E-E')}}{A\alpha}\right) \exp\left(\frac{2M_A(E-E')}{A\alpha}\right)\left(\frac{d\sigma}{dt\,dE}\right)_{\text{uncor.}}$$

Here $E = E_0 + t/2M_A$ and M_A is the mass of the target.

A detailed derivation of relations (5) and (6), together with the appendix of the obtained results for the calculations of cross sections for specific processes will be published separately.

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