A new class of nonlinear surface waves

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Exact solutions of Maxwell's equations in the form of nonlinear P-type surface waves are obtained for the boundary between two nonlinear uniaxial media '+' and '−' ($z>0$ and $z<0$, respectively), which are characterized by diagonal dielectric-constant tensors $\epsilon_{ij}^{\pm}(\omega,|E^{\pm}|^2)$ that depend quadratically on the amplitude of the electric field.

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We examine the possibility of the existence of surface modes at the boundary between two nonlinear media that fill the half-spaces $z>0$ and $z<0$, respectively. To simplify the calculations, we shall restrict our analysis to crystals with uniaxial symmetry, for which the dielectric-constant (DC) tensors are diagonal, but which have a strongly nonlinear amplitude of the electric field

$$\epsilon_{11}^{\pm} = \epsilon_{22}^{\pm} = \epsilon_{\pm}^{o}(\omega) + a_{\pm}(\omega)\,[\,|E_1^{\pm}|^2 + |E_2^{\pm}|^2\,];\ \epsilon_{33}^{\pm} = \epsilon_{\pm}(\omega).$$

The simplest physical mechanisms that lead to such nonlinearity are the Kerr optical effect, electrostriction, ionization of the medium by the wave field, etc., when an allowance for the first nonvanishing term of the expansion of the nonlinear part of the polarization in powers of the field amplitude leads to nonlinear processes at the fundamental frequency.

Maxwell's equations are valid in both half-spaces, including the boundary, and for the quasimonochromatic waves[1]

$$E_{1,3}^{\pm}(x,z,t) = \mathscr{E}_{1,3}^{\pm}(z,t)\exp\{i(qx-\omega t)\};\ E_2^{\pm}(x,z,t) = 0,$$
$$H_2^{\pm}(x,z,t) = H^{\pm}(z,t)\exp\{i(qx-\omega t)\};\ H_{1,3}^{\pm}(x,z,t) = 0 \qquad (1)$$

they have the following form

$$\frac{\partial H^{\pm}}{\partial z} = i\frac{\omega}{c}D_1^{\pm};\ qH^{\pm} = -\frac{\omega}{c}D_3^{\pm};\ \frac{\partial \mathscr{E}_1^{\pm}}{\partial z} - iq\mathscr{E}_3^{\pm} = i\frac{\omega}{c}H^{\pm};\ \mathbf{D} = \hat{\epsilon}\mathbf{E}. \qquad (2)$$

It is assumed that for a wave propagating along the x axis all the amplitudes $\mathscr{E}_{1,3}^{\pm}$ and $H^{\pm}$ vary slowly with the time t compared to the phase factor $\exp\{-i\omega t\}$, i.e., the physical processes that lead to the nonlinearity occur quite rapidly during the variation time of the field amplitudes. By eliminating the quantities $\mathscr{E}_3^{\pm}(z,t)$ and $H^{\pm}(z,t)$ from Eq. (2), we obtain for the amplitudes $\mathscr{E}_1^{\pm}(z,t)$ the equations

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$$\left[\frac{\partial^2}{\partial z^2} - k_\pm^2\,\epsilon_{11}^\pm/\epsilon_\pm\right]\mathcal{E}_1^\pm = 0, \tag{3}$$

which have the first "energy integrals"

$$\frac{1}{2}\left(\frac{\partial \mathcal{E}_1^\pm}{\partial z}\right)^2 - \frac{k_\pm^2}{2\epsilon_\pm}\left(\epsilon_\pm^0\,\mathcal{E}_1^{\pm 2} + a_\pm\,\mathcal{E}_1^{\pm 4}/2\right) = C_\pm\,, \tag{4}$$

where $k_\pm^2 = q^2 - \omega^2\epsilon_\pm/c^2$, and the constants $C_\pm = 0$ because the sought-for solutions vanish at infinity ($\pm\infty$). This makes it possible to find simple solutions of Eqs. (4)

$$\mathcal{E}_1^\pm(z) = (-2\epsilon_\pm^0/a_\pm)^{1/2}\cosh^{-1}[(\epsilon_\pm^0/\epsilon_\pm)^{1/2}\,k_\pm(z - z_\pm^0)], \tag{5}$$

which satisfy the original equations (3). The unknown parameters $z_\pm^0$, which define the initial "phases" with respect to the z coordinate, are expressed in terms of the boundary value of the tangential component of the electric-field intensity in accordance with the continuity boundary condition at the boundary for $z = 0$

$$\left(-\frac{2\epsilon^0}{a_-}\right)^{1/2}\cosh^{-1}\left[\left(\frac{\epsilon_-^0}{\epsilon_-}\right)^{1/2} k_- z_-^0\right] =$$

$$\mathcal{E}_1(0) = \left(-\frac{2\epsilon_+^0}{a_+}\right)^{1/2}\cosh^{-1}\left[\left(\frac{\epsilon_+^0}{\epsilon_+}\right)^{1/2} k_+ z_+^0\right], \tag{6}$$

which establishes the relationship between z_+^0 and z_-^0.

The continuity boundary condition of the tangential component of the magnetic field at the boundary between the media, which, according to Eq. (2), has the form

$$H^\pm(z) = i\,(\omega/c\,k_\pm)\,\partial\mathcal{E}_1^\pm(z)/\partial z\,, \tag{7}$$

makes it possible to find the nonlinear dispersion equation for the surface modes $\omega = \omega[q, \mathcal{E}_1(0)]$

$$(\epsilon_-^0\epsilon_-)^{1/2}k_-^{-1}\tanh[(\epsilon_-^0/\epsilon_-)^{1/2}k_- z_-^0] = (\epsilon_+^0\epsilon_+)^{1/2}k_+^{-1}\tanh[(\epsilon_+^0/\epsilon_+)^{1/2}k_+ z_+^0]. \tag{8}$$

In particular, it follows from this that the parameters z_+^0 and z_-^0 have identical signs: $z_+^0 z_-^0 > 0$. If they are positive, then, according to the solutions (5), the amplitudes $\mathcal{E}_1^\pm(z)$ increase initially from $\mathcal{E}_1(0)$ to their maximum values at $z = z_\pm^0$, and then they decrease to zero at infinity. For negative values of $z_\pm^0$ the wave amplitudes decrease immediately from the boundary value $\mathcal{E}_1(0)$ to zero as $z \to \pm\infty$.

By using the relation (6) we can transform the dispersion equation (8) into an expression for the index of refraction

$$n^2(\omega, \mathscr{E}_1(0)) = \left(\frac{qc}{\omega}\right)^2 = \frac{\epsilon_+\epsilon_-[\epsilon_+^0 - \epsilon_-^0 + (\alpha_+ - \alpha_-)\mathscr{E}_1^2(0)/2]}{\epsilon_+\epsilon_+^0 - \epsilon_-\epsilon_-^0 + (\alpha_+\epsilon_+ - \alpha_-\epsilon_-)\mathscr{E}_1^2(0)/2} \cdot \quad (9)$$

The requirement that it must be real, $n^2(\omega, \mathscr{E}_1) > 0$, gives rise to the restrictions on the value of $\mathscr{E}_1(0)$ in the form of the inequalities

$$\begin{cases} (\alpha_+\epsilon_+ - \alpha_-\epsilon_-)\mathscr{E}_1^2(0) \lessgtr 2(\epsilon_-\epsilon_-^0 - \epsilon_+\epsilon_+^0); \\ (\alpha_+ - \alpha_-)\mathscr{E}_1^2(0) \begin{Bmatrix} \lessgtr, \\ \gtrless \end{Bmatrix} 2(\epsilon_-^0 - \epsilon_+^0); \quad \epsilon_+\epsilon_- \begin{Bmatrix} > \\ <, \end{Bmatrix} 0. \end{cases} \quad (10)$$

The condition that the amplitudes of the solutions (5) and their zero asymptotic form at infinity (along the z coordinate) must be real impose certain relationships between the signs of the parameters of the medium: $\alpha_\pm(\omega)\epsilon_\pm^0(\omega) < 0$ and $\epsilon_\pm^0(\omega)\epsilon_\pm(\omega) > 0$, according to which the obtained nonlinear modes can exist in those frequency domains in which linear surface waves are not realized,[2] for example, for $\alpha_\pm(\omega) < 0$ and $\epsilon_\pm^0(\omega)$, $\epsilon_\pm(\omega) > 0$ or $\alpha_\pm(\omega) > 0$ and $\epsilon_\pm^0(\omega)$, $\epsilon_\pm(\omega) < 0$. It should also be noted that in order to make a valid choice of the DC in the indicated form, we must be sure that $\mathscr{E}_{\max}^{\pm}(z_\pm^0)$ is small compared to the intracrystalline fields, i.e., that $(-2\epsilon_\pm^0/\alpha_\pm) \ll 1$; this can be accomplished for small $\epsilon_\pm^0(\omega)$ or large $\alpha_\pm(\omega)$. Otherwise, we must use the more general form of the material nonlinear relations.

Using the solutions (5) and relations (6) and (8), we can easily switch to the limiting case of the boundary between linear and nonlinear media, which was examined in Ref. 3, and also to solutions in the form of linear surface modes at the boundary between two linear media.[2] Assuming that the quantity $\mathscr{E}_1(0)$ is finite, we can set one of the coefficients $\alpha_\pm$ (or both simultaneously) to zero. In this case, as follows from Eq. (6), one of the parameters $z_\pm^0$ (or both simultaneously) must approach $\pm\infty$, respectively. As a result, the dispersion equation (8) will become either a dispersion equation of nonlinear surface polaritons or a dispersion law of linear surface waves. The wave amplitudes (5) will become a solution for nonlinear polaritons or a solution of linear surface modes that decreases exponentially at the boundary.

These limiting transitions occur more clearly in the expression (9) as $\alpha_\pm$ approaches zero successively or simultaneously.

In conclusion, we note that the determined class of exact solutions of the nonlinear equations (3) is not the only possible one, since these equations allow separate solutions of the solitary-wave type as well as periodic (along the x axis) solutions,[1,4] which require an independent analysis.

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