Ferromagnetism and charge-density waves in ionized gases

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An electron–neutral interaction in a cool plasma can lead to a ferromagnetic phase transition, the onset of a periodic structure (a charge-density wave), and an instability of an electron beam in a neutral gas with respect to the growth of some characteristic high-frequency waves. Two ESR frequencies are calculated.

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In an ionized gas in which the degree of ionization is not too high the free energy may be affected substantially by not only the correlation effects associated with the Coulomb interactions of ions and electrons but also the virial corrections resulting from the interactions of charged particles with neutrals and of neutral particles with other neutrals. In neutral gases (except cryogenic gases[1]) these corrections do not usually lead to any qualitatively new phenomena, because the gases condense before the quantum properties become noticeable. In contrast, for collisions for (light) electrons with neutrals the characteristic energy $E_0 = \hbar^2/\mu r_0^2$ (μ is the reduced mass and r_0 is the interaction radius) turns out to be of the order of 10^5 K, so that quantum effects may be manifested over a much broader range.

1. Since the de Broglie wavelength of an electron, λ_e, is much longer than those of neutrals or ions, the thermodynamics of an ionized gas is determined primarily by (in addition to the correlation terms) the virial corrections associated with the mutual scattering of electrons and neutrals. For clarity, we shall discuss here the results found in only the lowest-order approximation in r_0/λ_e, although some of the results could easily be expressed in terms of the exact amplitude for electron scattering by a neutral atom. If the neutral has an electron spin, say spin 1/2 for definiteness, the electron scattering length can be written as follows:

$$a = (a_1 + a_2 \vec{\sigma}_e \vec{\sigma}_n)/4, \quad a_1 = 3a_- + a_+, \quad a_2 = a_- - a_+ \tag{1}$$

where a_- and a_+ correspond to triplet and singlet scattering; the subscripts "e," "n," and "i" denote electrons, neutrals, and ions; and the $\vec{\sigma}$ are the Pauli matrices. If the spin systems of the electrons and neutrals are polarized, then the virial correction to the free energy of the plasma is

$$F_{vir} = (\pi\hbar^2 N_e N_n/2\mu)(a_1 + a_2 \alpha_e \alpha_n \mathcal{M}_e \mathcal{M}_n). \tag{2}$$

Here $N_{e,n}$ are the number densities of the particles;

$$\alpha_{e,n} = (N_{e,n}^{\uparrow} - N_{e,n}^{\downarrow})/N_{e,n}, \quad N_{e,n}^{\uparrow} + N_{e,n}^{\downarrow} = N_{e,n},$$

are the degrees of polarization; and $\mathcal{M}_{e,n}$ are the directions of the magnetization vectors for the two components. Using (2), and minimizing the total free energy with respect to a_e and a_n (we are also ignoring the small electron–electron exchange corrections), we find the condition for the appearance of magnetic order, $\theta \leqslant \theta_{c2}$,

$$\theta_{c2}^2 = E_2^2 (N_e |a_2|^3)(N_n |a_2|^3), \quad E_2 = \pi \hbar^2 / 2\mu a_2^2, \quad \theta^2 = T_e T_n, \tag{3}$$

and the equilibrium values of a_e and a_n,

$$a_e = 3^{1/2} (\theta_{c2}^2/\theta^2 - 1)^{1/2} [1 + c (T_e^2/\theta_{c2}^2)]^{-1/2}, \quad c = N_e/N_n$$

$$a_n = 3^{1/2} (\theta_{c2}^2/\theta^2 - 1)^{1/2} [1 + c^{-1} (T_n^2/\theta_{c2}^2)]^{-1/2}, \quad N_e^{1/3} |a_2| \gg c^{1/2} \gg$$

$$\gg (m_e/m_n)(N_n^{1/3} |a_2|)^{-1} \tag{4}$$

where $T_{e,n}$ are the temperatures of the electrons and the neutrals. If $a_2 > 0$, the spontaneous magnetic moments of the electrons and neutral atoms are antiparallel, $\mathcal{M}_e \mathcal{M}_n = -1$, while if $a_2 < 0$ they are parallel, $\mathcal{M}_e \mathcal{M}_n = 1$. As the temperature is lowered, the interaction of neutrals and electrons can also lead to a spontaneous increase or decrease in the density of each component; i.e., it can lead to a disruption of the electrical neutrality of the plasma in regions of macroscopic size. On the other hand, such a redistribution of electrons would disrupt the homogeneity of the plasma and lead to a macroscopic electric field, so that the correlation energy of the electrons and ions would increase. These two competing mechanisms cause a phase transition in the system which is associated with the appearance of a periodic structure in distribution of the electric field and of the densities of all the particle species: charge-density waves. The electric potential ϕ and the perturbations of the densities, δN, satisfy the equations

$$N_i + \delta N_i = N_i \exp(-ze\phi/T_i), \quad N_e + \delta N_e = N_e \exp[(e\phi/T_e) - (g_1 \delta N_n/T_e)],$$

$$N_n + \delta N_n = N_n \exp(-g_1 \delta N_e/T_n), \quad \Delta\phi + 4\pi e (z \delta N_i - \delta N_e) = 0, \tag{5}$$

where z is the ion charge, and $g_1 = \pi h^2 a_1/\mu$. Linearizing (5), and eliminating $\delta N_{e,n,i}$, we find

$$\Delta\phi - [d_i^{-2} + d_e^{-2}(1 - G)]\phi = 0, \quad G = g_1^2 N_e N_n / T_e T_n \tag{6}$$

where $d_{e,i}$ are the electron and ion Debye lengths. Equation (6) determines the screening length of the electric field in the ionized gas, D,

$$D^{-2} = d_i^{-2} + d_e^{-2}(1 - G), \tag{7}$$

and if

$$\theta_{c1}^2 > \theta^2 > \theta_{c1}^2 \left(1 + \frac{T_i}{zT_e}\right)^{-1}, \text{ where } \theta_{c1}^2 = E_1^2 (N_e |a_1|^3)(N_n |a_1|^3) \tag{8}$$

$$\text{and } E_1 = \pi \hbar^2 / 2\mu a_1^2,$$

this equation has an oscillatory solution which corresponds to the appearance of charge-density waves of length $2\pi|D|$. Equations (3) and (8) are essentially equations for the phase-transition temperature, since N_e and N_i in the plasma are functions of the temperature (and of the external conditions). It appears that such transitions could actually be observed at sufficiently high pressures and with a large difference between the electron and ion temperatures. Analogous effects can occur in solid solutions and heavily doped metals and semiconductors.

2. At temperatures $T \ll \hbar^2/\mu r_0^2$ the interaction of electrons with neutrals should also be incorporated in the kinetic equations for the ionized gas. The result is the appearance of a corresponding quantum correction in the kinematic part of the equations, along with the self-consistent Coulomb field. This correction is linear in the forward-scattering amplitude.[1] In the present case, for longitudinal waves in the collisionless limit, we have

$$(\omega - \mathbf{k}\mathbf{v}_e)\,\delta n_e + \frac{4\pi e^2}{k^2}\,\mathbf{k}\,\frac{\partial n_e^{(0)}}{\partial \mathbf{p}}\,\Big[\sum_{\mathbf{p}}(\delta n_e + z\,\delta n_i)\Big] + \mathbf{k}\,\frac{\partial n_e^{(0)}}{\partial \mathbf{p}}\,g_1 \sum_{\mathbf{p}} \delta n_n = 0$$

$$(\omega - \mathbf{k}\mathbf{v}_i)\,\delta n_i + \frac{4\pi z e^2}{k^2}\,\mathbf{k}\,\frac{\partial n_i^{(0)}}{\partial \mathbf{p}}\,\Big[\sum_{\mathbf{p}}(\delta n_e + z\,\delta n_i)\Big] = 0, \tag{9}$$

$$(\omega - \mathbf{k}\mathbf{v}_n)\,\delta n_n + \mathbf{k}\,\frac{\partial n_n^{(0)}}{\partial \mathbf{p}}\,g_1 \sum_{\mathbf{p}} \delta n_e = 0,$$

where ω and $\mathbf{k}$ are the frequency and wave vector of the waves, $v_{n,e,i}$ are the velocities of the particles, and $m_{n,e}$ are the masses of the neutrals and electrons. For the spectrum of ion-acoustic waves, $kv_{Te} \gg \omega \gg kv_{Tn}, kv_{Ti}$, for example, we find from (9)

$$\omega^2 = \Omega_i^2\,[1 + (kd_e)^{-2}]^{-1}\,[3(kd_i)^2 + 3\,(d_i/d_e)^2 + G\,(d_n/d_e)^2],\quad d_n = v_{Tn}/\Omega_i, \tag{10}$$

where v_T is the thermal velocity, and Ω_i is the ion plasma frequency. In the static limit, $\omega \ll kv_{Te}, kv_{Tn}, kv_{Ti}$, Eqs. (9) are equivalent to the dispersion relation

$$1 + (kd_e)^{-2} + (kd_i)^{-2} - G\,[1 + (kd_i)^{-2}] = 0, \tag{11}$$

which agrees completely with (7) and (8). Under certain special conditions, some distinctive high-frequency waves (similar to zero sound or spin waves in a Fermi liquid[2]) can propagate, as a result of the mutual zero-angle scattering of electrons and neutrals. Let us assume that a monoenergetic beam of slow electrons of rather low density (so that collisions of electrons with gas molecules are predominant) is moving at a velocity $\mathbf{u}$ through a neutral gas at rest. From the kinetic equations for the beam and gas particles analogous to Eqs. (9), we can determine the dispersion law for high-frequency ($|\omega| \gg kv_T$) density waves of the neutral molecules and electrons:

$$\omega = \mathbf{k}\mathbf{u}/2 \pm [(\mathbf{k}\mathbf{u})^2/4 \pm s_0^2 k^2]^{1/2},\quad s_0^2 = |g_1|\,(N_e N_n/m_e m_n)^{1/2}. \tag{12}$$

If $u/v_{Tn} \gg 1$, expression (12) clearly holds near $\omega = ku$, but if $s_0 \gg v_{Tn}$ then it holds for arbitrary values of the scalar product **ku**, and the beam–gas system is unstable with respect to one of the branches in (12) for $|\cos\chi| < 2s_0/u$, where χ is the angle between **k** and **u**. If $s_0 \geqslant u/2$, the instability occurs for all values of χ. If the gas molecules have an electron spin, then magnetization waves with a spectrum similar to (12) (a_1 is replaced by a_2) may be excited and may grow in the beam–gas system. In a ferromagnetically ordered plasma, there is also the possibility that spin waves with a quadratic dispersion law will propagate.[1] The corresponding results will be published elsewhere. Here we will give the results for only homogeneous magnetic oscillations of a weakly ionized gas in an external magnetic field **H**. There are two solutions for an electron spin resonance, one of which,

$$\omega = 2\beta H/\hbar \tag{13}$$

corresponds to an ordinary uniform precession of the total magnetic atom of electrons and neutrals (β is the electron magnetic moment) and the other

$$\omega = (2\beta H/\hbar) - (2\pi a_2\hbar/\mu)(N_e + N_n)\,\mathrm{th}(\beta H/T), \tag{14}$$

describes a precession of the two magnetic moment vectors, equal in magnitude but opposite in direction, of the two subsystems. For cesium in a magnetic field $H \leqslant 100$ kOe, the observation of resonance (14) would be experimentally feasible at $T \geqslant 10^3$ K.

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