Oscillation of a stack of disks and friction between the normal and superfluid components in rotating superfluid ^{3}He

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(Submitted 24 March 1981; resubmitted 25 June 1981)

Pis'ma Zh. Eksp. Teor. Fiz. **34**, 97–101 (5 August 1981)

The parameters of the friction force between the normal and superfluid components in rotating ^{3}He can be determined in an Andronikashvili experiment with an oscillating stack of disks. These parameters are quite different for the *A* and *B* phases, and their measurement should yield useful information about the textures which arise during rotation.

PACS numbers: 67.50. – b, 67.90. + z

The friction force acting between the normal and superfluid components in rotating He II is usually determined from second-sound experiments.[1] This force should also affect the oscillation frequency of objects immersed in He II, since it results in an entrainment of the superfluid component. In this case, however, there is a more effective mechanism for the entrainment of the superfluid component—the pinning of vortices—so that the friction force usually amounts to a second-order correction.[2,3] The situation is quite different in superfluid ^{3}He: Because of the far lower temperatures, the mean free path and the viscosity coefficients are several orders of magnitude larger than in He II. Since the second-sound velocity is also low, it is extremely difficult to observe second sound.[1)] On the other hand, the bending modulus of the vortices (or of the inhomogeneous texture in the rotating *A* phase; more on this below) is comparable to or slightly smaller than, that in He II. Consequently, the displacements of vortices caused by the pinning penetrate a distance $l_{\text{bend}} \sim \sqrt{\nu_{\text{bend}}/\omega}$ which is small in comparison with the viscous penetration depth $l_\nu \sim \sqrt{\nu_\nu/\omega}$. Here $\nu_{\text{bend}} \sim \hbar/m$ and ν_ν are the bending modulus of the vortices and the kinematic viscosity, respectively, and ω is a characteristic frequency: a combination of the oscillation and revolution frequencies, which are usually comparable in magnitude. It follows that it is always possible to arrange an experiment with a stack of disks[1)] which are separated by a distance much greater than l_{bend} but much smaller than l_ν, so that the entire normal part of the liquid between the disks is entrained by the disks and moves at their velocity. The velocity of the vortices and of the superfluid component, on the other hand, does not vary along the oscillation axis of the disks, which coincides with the rotation axis. The latter assumption is valid except in thin regions of thickness $\sim l_{\text{bend}}$ near the surfaces of the disks, which make only a small contribution to the moment of inertia of the liquid, especially since the vortex pinning in ^{3}He should apparently be weaker than in He II (because the scale dimensions of the inhomogeneities of the order-parameter field in ^{3}He are significantly larger).

For both the A and B phases we can use the results of a general analysis of the problem of the oscillation of disks in He II (Refs. 2 and 3). In the simple case under consideration here, in which all the velocities remain constant along the rotation axis (which is also the oscillation axis), i.e., if pinning is ignored, we find the following for the disk oscillation frequency ω:

$$\omega^2 = \frac{\omega_0^2}{1 - \gamma \dfrac{\rho_s}{\rho}(1+g)}, \qquad \gamma = \frac{I_{\mathrm{liq}}}{I_{\mathrm{liq}} + I_d}, \tag{1}$$

where ω_0 is the oscillation frequency of the stack of disks in the normal liquid above the critical point, and I_d and I_{liq} are the moments of inertia of the disks and of the liquid between the disks. The quantity g, which describes the entrainment of the superfluid component by the friction force, is given by

$$g = \frac{\dfrac{\rho_n}{2\rho} B'\left(1 - \dfrac{\rho_n}{2\rho} B'\right) - \dfrac{\rho_n}{2\rho} B\left(\dfrac{i\omega}{2\Omega} + \dfrac{\rho_n}{2\rho} B\right)}{\left(1 - \dfrac{\rho_n}{2\rho} B'\right)^2 + \left(\dfrac{i\omega}{2\Omega} + \dfrac{\rho_n}{2\rho} B\right)^2}, \tag{2}$$

where Ω is the angular velocity of the rotation, B and B' are the Hall and Vinen parameters, which determine the vortex velocity $\mathbf{v}_L$ and the friction force,

$$\mathbf{v}_L = \mathbf{v}_s + \frac{\rho_n}{2\rho}\left(B'\Omega(\mathbf{v}_n - \mathbf{v}_s) - B[\vec{\Omega} \times (\mathbf{v}_n - \mathbf{v}_s)]\right), \tag{3}$$

and $\mathbf{v}_n$ and $\mathbf{v}_s$ are the normal and superfluid velocities. If there is no friction force ($B=B'=0$), the rotation obviously has no effect ($g=0$), and the disks entrain only the normal part of the liquid in their oscillation. In the opposite limit of a large friction force ($B, B' \to \infty$), the superfluid part is also completely entrained ($g=-1$), and there is no change in the oscillation frequency when the critical point is crossed in the rotating liquid.

The applicability of Eqs. (1)-(3) for the B phase, in which (as in He II) an array of singular vortices arises during the rotation, is quite obvious; only for the A phase is an explanation required. The derivation of Eq. (1) is based on the following equation of motion for the superfluid component:

$$\frac{d\mathbf{v}_s}{dt} + \nabla\left(\mu + \frac{v_s^2}{2}\right) + [\mathrm{rot}\, \mathbf{v}_s \times \mathbf{v}_L] = 0. \tag{4}$$

After an average is taken over the inhomogeneous texture, $\mathrm{rot}\mathbf{v}_s$ ($\mathrm{curl}\mathbf{v}_s$) should be replaced by 2Ω Equation (4) is obvious in the presence of a magnetic field, in which case there are isolated vortices moving at $\mathbf{v}_L$. In a zero magnetic field, in which there is a continuous periodic texture, the velocity $\mathbf{v}_L$ is the displacement velocity of the texture, so that $dl/dt = -(\mathbf{v}_L \nabla)\mathbf{1}$. A change in $\mathbf{1}$, however, leads to a change in $\mathbf{v}_s$:

$$\left.\frac{dv_{si}}{dt}\right|_{l} = -[\mathbf{l} \times \nabla_i \mathbf{l}]\frac{d\mathbf{l}}{dt} = [\mathbf{l} \times \nabla_i \mathbf{l}](\mathbf{v}_L \nabla)\mathbf{l}. \tag{5}$$

Using the Ho–Mermin relation[6]

$$(\mathrm{rot}\ \mathbf{v}_s)_i = \frac{1}{2}\epsilon_{ijk}\ \mathbf{l}[\nabla_j \mathbf{l} \times \nabla_k \mathbf{l}], \tag{6}$$

we find that the change in the velocity $\mathbf{v}_s$ associated with the motion of the texture is equal to the vector product in Eq. (4).

What values of B and B' should we expect in the region corresponding to the Ginzburg–Landau theory? In the B phase the mean free path of the quasiparticles is always larger than the vortex core, except in a small neighborhood of the critical point. We can thus apply the theory of Hall and Vinen, which is based in turn on the theory for the scattering of noninteracting quasiparticles.[7] According to this theory, we would have $B \to 0$ and $B' \to 2$ in the limit $\rho_s \to 0$ (Ref. 1). It then follows from (3) that the vortices move at the normal velocity, i.e., $\mathbf{v}_L = \mathbf{v}_n$. This result can be explained as follows: The vortex velocity must be determined by the equilibrium between the force acting between the vortex and the superfluid component, i.e., the Magnus force, and the friction force between the normal component and the vortex, which is proportional to the relative velocity $\mathbf{v}_n$-$\mathbf{v}_L$. The proportionality coefficient for that component of the force which is transverse with respect to the velocity $\mathbf{v}_n$-$\mathbf{v}_L$ falls off as $\sqrt{\rho_s}$ in the limit $\rho_s \to 0$ according to calculations in the BCS model for type II superconductors,[8,9] whereas the Magnus force is proportional to ρ_s. We thus have $\mathbf{v}_L = \mathbf{v}_n$ in the limit $\rho_s \to 0$. We see from Eqs. (1) and (2) that the rotation has no effect on the oscillation frequency in the B phase. Consequently, when the line of the second-order phase transition is crossed directly into the B phase (i.e., at low pressures, below the tricritical point), the oscillation frequency of the stack of disks will increase in proportion to ρ_s, as in ^{3}He at rest.

In the A phase, in a sufficiently weak magnetic field, an array of nonsingular vortices arises,[10,11] with a core much larger than the mean free path; in this case we can use the results calculated by Kopnin[12] for the velocity of an isolated nonsingular vortex in the A phase. It follows from this calculation that the friction force in the A phase is determined by the orbital viscosity, that $B = 0$, and that the parameter $B' \sim 1/(T_c - T)^{3/2}$ is quite large in the Ginzburg–Landau region. As a result, we find that the superfluid component is completely entrained by the normal component. On the line of the second-order phase transition between the normal ^{3}He and the A phase, therefore, there should not be any significant change in the disk oscillation frequency. With a further temperature decrease on the line of the first-order phase transition to the B phase, there should be an abrupt increase in the oscillation frequency.

In discussing the A phase we have ignored that component of the superfluid current which results from the gradients of $\mathbf{l}$. This component determines the spontaneous orbital angular momentum, but it is important only at low rotation velocities, $\Omega \lesssim \hbar/mR^2$, where R is the radius of the disks.[13] For a slow rotation the friction force and the oscillation frequency may depend on the initial texture of $\mathbf{l}$, for example, on whether **l** is parallel or antiparallel to the angular velocity, and measurements with a varying angular velocity could, in principle, reveal hysteresis effects demonstrating the existence of a spontaneous angular momentum in the A phase.

As the magnetic field is strengthened, the core size of a nonsingular vortex decreases,[14] becoming comparable to the mean free path. With smaller cores, the effect of the rotation on the oscillation frequency disappears. For singular vortices, on the other hand, the core is always smaller than the mean free path, and the magnetic field does not affect the oscillation frequency. From the behavior of the oscillation frequency of a stack of disks in a magnetic field we can thus draw conclusions regarding the type of texture that arises whether it is an (equilibrium or metastable texture).

I wish to thank G. E. Volovik and N. B. Kopnin for useful discussions.

1)There is the hope, however, that the friction force can be measured in steady-state heat fluxes in rotating vessels, by measurements similar to those carried out in He II by Yarmchuk and Glauberson.[4]

2)Experiments of this type have already been carried out[5] in ^{3}He at rest.

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Translated by Dave Parsons
Edited by S. J. Amoretty