Localized superconductivity in *N*-*S*-*N* systems

A. I. Buzdin and L. N. Bulaevskiĭ

P. N. Lebedev Physics Institute, Academy of Sciences of the USSR

(Submitted 23 June 1981)

Pis'ma Zh. Eksp. Teor. Fiz. **34**, No. 3, 118–121 (5 August 1981)

The superconductivity localized near a film has been detected at a temperature T_c slightly higher than T_{c0} in systems comprised of two massive superconductors with T_{c0}, which are divided by a thin film with $T_{cl} > T_{c0}$. The temperature T_c, the upper critical field, and the diamagnetic moment have been determined.

PACS numbers: 73.60.Ka, 74.10. + v

The detection of superconductivity of the twinning plane of a tin crystal in the temperature region slightly higher than the critical temperature T_{co} of the bulk metal was recently reported in Ref. 1. In view of this, we shall analyze a model of two bulk superconductors with a critical temperature T_{co}, which are separated by a thin superconducting layer S of thickness $d \ll \xi_0$ with a critical temperature $T_{cl} > T_{co}$ ($\xi_0 = 0.18\, v_F/T_{co}$ is the coherence length). We believe that such a model adequately describes the effect of plane microscopic defects on the superconductivity (for example, the twinning plane[1]), as well as of the N–S system obtained by sputtering. We shall therefore analyze a case which is opposite to that in Ref. 2, where the effect of extended defects with $d \gg \xi_0$ on the superconductivity was studied.

1. To determine the critical temperature and the incipient superconductivity, we shall use the integral equation for the order parameter Δ. We assume that the Cooper pairing constant λ depends only on the coordinate: $x : \lambda = \lambda_0 + \lambda_1(x)$, λ_0 is the corresponding constant for the bulk metal, and $\lambda_1(x)$ describes the increase of Cooper pairing in the film S (yz plane); $\lambda_1(x)$ is localized in a narrow region near $x = 0$. In our case it is more convenient to write the integral equation for $\psi(x) = \Delta(x)/\lambda(x)$

$$\psi(x) = \lambda_o \int K(r - r')\, \psi(x')\, dr' + \int K(r - r')\, \lambda_1(x')\, \psi(x')\, dr'. \tag{1}$$

$K(r)$ is the superconducting kernel (see, for example, Ref. 3). Since $\lambda_1(x)$ is localized in a narrow region $d \ll \xi_0$, we solve Eq. (1) by converting it to a momentum representation.

Thus we can determine the increase of the critical temperature from a comparison with the bulk superconductor

$$\tau_o = (T_c - T_{co})/T_{co} = \bar{\lambda}_1^2\, d^2\, T_{co}^2 / 4\beta \lambda_o^4\, v_F^2 \sim (10/\lambda_o^2)(\lambda_1 d/\lambda_o \xi_o)^2,$$

where $\beta \approx 0.02$ and $\bar{\lambda}_1 d = \int \lambda_1(x)\, dx$. In the case of a dirty superconductor, $l \ll \xi_0$, an additional multiplier $\sim \xi_0/l \gg 1$ appears in the expressions for τ_0. Note also that the increase of T_c near a filamentary defect is exponentially small $\tau_0 \sim \exp(-\lambda_1 d^2/\lambda_0^2 \xi_0^2)$ and is missing entirely in the case of a point defect. The obtained results show that even if the critical temperature of the film S is much higher than that of the bulk sample T_{co}, the critical temperature $T_c = (1+\tau_0)\,T_{co}$ is only slightly higher than T_{co} because of proximity effects. (Note that if the critical temperature of a bulk superconductor $T_{co} \to 0$, then $\tau_0 \to 0$ and the effect under consideration vanishes.) It follows from Eq. (1) that the appearing superconductivity has a localized nature: $\psi(x) \sim \exp[-|x|/\xi(T)]$.

2. Analyzing the behavior of the order parameter Δ on the scale $r \gg \xi$, we obtain the following equation for $\Delta(r)$:

$$\left[\frac{1}{4m}\frac{\partial^2}{\partial r^2} + \frac{1}{\eta}\frac{T_{co}-T}{T_{co}} - \frac{3}{4}\frac{\Delta^2(r)}{\epsilon_F}\right]\Delta(r) = -\gamma\delta(x)\,\Delta(r) \tag{2}$$

$$\eta = 7\zeta(3)\,\epsilon_F/6(\pi T_{co})^2, \quad \gamma = (\tau_o/\eta m)^{1/2}.$$

The solution of Eq. (4) is

$$\Delta^2(x) = \frac{32\,\epsilon_F(\tau_o - \tau)\exp[-2|x|/\xi(\tau)]}{3\eta\{1+(\tau_o/\tau)^{1/2} + [1-(\tau_o/\tau)^{1/2}]\exp[-2|x|/\xi(\tau)]\}} \tag{3}$$

$$\xi^1(\tau) = \eta/4m\tau = 0{,}55\,\xi_o^2\,\tau^{-1}, \quad \tau = (T - T_{co})/T_{co}$$

3. Determining the behavior of the order parameter Δ from Eq. (5), we can solve the problem of screening the weak magnetic field in our system. For a field parallel to the film S (parallel to the yz plane) $(\tau - \tau_0)/\tau_0 \ll 1$ in the limit. We find that

$$B(-x) = B(x) = H_o\,u\,[K_1(u) + K_o(k)\,I_1(u)/I_o(k)]$$

$$u = k\exp(-x/\xi), \quad x > 0, \tag{4}$$

where $k = 3\xi_0(\tau_0-\tau)^{1/2}/\lambda_L(0)\,(\tau^{1/2}+\tau_0^{1/2})$. For $k \ll 1$,

$$B(x) = H_o\left[1 - \frac{\tau_o - \tau}{\tau_o}\;0{,}55\;\frac{\xi_o^2}{\lambda_L^2(0)}\left(1 + \frac{2x}{\xi}\right)\exp(-2x/\xi)\right]. \tag{5}$$

For $k \gg 1$ the field near $x = 0$ is screened out almost completely and $B(0) = H_0(\pi k/2)^{1/2}\exp(-k)$. The diamagnetic moment per unit area S is

$$-M = \int\frac{H_o - B}{4\pi}\,dx = \begin{cases}\xi H_o k^2/8\pi & \text{as } k \to 0\\ \xi H_o[\ln(k/2)+\gamma]/2\pi & \text{as } k \to \infty, \quad \gamma = 0{,}57\end{cases} \tag{6}$$

It follows from Eq. (6) that the characteristic screening length for a type I supercon-

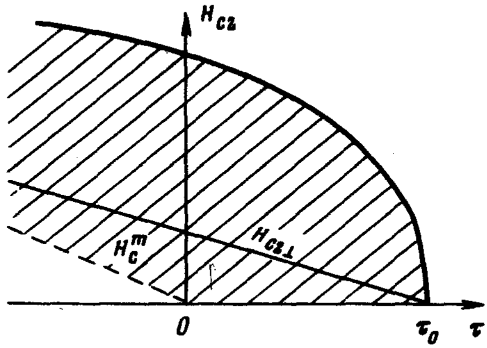

FIG. 1. Temperature dependence of the upper critical fields $H_{c2\perp}$ and $H_{c2\parallel}$. H_{c2} is in units of Φ_0/ξ_2^0; the dashed line shows schematically the temperature dependence of the critical field H_c^m in the massive sample. The hatched area represents the region of localized superconductivity.

ductor is of the order of $\xi_0\tau^{-112}$, consistent with the value of about 3.72 K obtained in Ref. 1. As we move away from T_{co}, the diamagnetic moment decreases according to the law $\tau^{-\alpha}$ $0.5<\alpha<1$, and the magnetic moment is proportional to (T_c-T) as $T\to T_c$. It was alleged in Ref. 1 that the magnetic moment decreases exponentially with increasing temperature, which is inconsistent with the model examined by us. The reason for this descrepancy is unclear. We believe that this problem can be clarified and the specific properties of tin crystals with a twinning plane can be understood by measuring the magnetic moment of tin on which a superconductor with a higher critical temperature has been sputtered.

4. The upper critical field, which is parallel to the layer, is determined by the expressions

$$H_{c2\parallel}(T\to T_c) = 0.85\,\tau_o\Phi_o\,[(T_c-T)/T_{co}]^{1/2}/\xi_o^2,$$

$$H_{c2\parallel}\;(T=T_{co}) = 1.15\;\Phi_o\tau_o/\xi_o^2\;, \qquad (7)$$

where $\Phi_0=\pi c\hbar/e$ is the flux quantum. As regards $H_{c2\perp}$, it has a standard temperature dependence: $H_{c2\perp}(T)=0.29\,(\Phi_0/\xi_0^2)\,(T_c-T)/T_{co}$.

The $H_{c2}(T)$ dependence is shown in Fig. 1. We point out that since the field H_{c2} is higher than the critical field H_c^m of the massive sample, there will be a localized superconductivity in the field even when $T<T_{co}$ in the fields $H_{c2}>H>H_c^m$.

The authors thank E. T. Maksimov and I. I. Khlyustikov for useful discussions.

1. M. S. Khaĭkin and I. N. Khlyustikov, Pis'ma Zh. Eksp. Teor. Fiz. 33, 167 (1981) [JETP Lett. 33, 158 (1981)].
2. V. M. Nabutovskiĭ and B. Ya. Shapiro, Zh. Eksp. Teor. Fiz. 75, 948 (1978) [Sov. Phys.–JETP 48, 480 (1978)].
3. A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Methods of Quantum Field Theory in Statistical Physics), Moscow, 1962.

Translated by S. J. Amoretty
Edited by Robert T. Beyer