Observation of a hot-hole population inversion in germanium

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An inversion of the distribution function of light holes with respect to that of heavy holes has been observed experimentally at energies corresponding to transitions between bands 1 and 2 in p-Ge ($\lambda = 50$–$150\ \mu$m) in strong crossed electric (**E**) and magnetic (**H**) fields at $T_0 = 18$ K and $H = 23$ kOe.

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The low-temperature scattering of charge carriers by optical phonons radically changes the energy distributions and galvanomagnetic effects in strong fields **E** and **H**. These changes were first studied theoretically by Vosilyus and Levinson.[1] Under certain conditions the energy distribution may become inverted, as has been shown by Monte Carlo numerical calculations[2] and analytic calculations.[3] From the standpoint of practical applications, the most interesting case is the inversion in p-type Ge of the energy distribution of light holes with respect to that of heavy holes (a population inversion), which may lead to an intensification of the far-infrared emission.[4]

Let us examine the reason for the population redistribution of the light holes. The trajectory traced out by holes in momentum space in crossed electric ($\mathbf{E} \parallel ox$) and magnetic ($\mathbf{H} \parallel oz$) fields is a circle which is oriented perpendicular to the p_z axis and whose center is displaced from the origin along the p_y axis:

$$p_x^2 + (p_y - p_H)^2 = p_{0_x}^2 + (p_{0_y} - p_H)^2, \quad p_z = \text{const};$$

where $p_H = m^* cE/H$, and p_{0_x} and p_{0_y} are the initial momentum components of the hole. We shall use the subscript "1" to refer to heavy holes and "2" to refer to light holes. We assume $\tau_1^-, \tau_2^- \gg \tau^+$ [τ^- is the characteristic time for hole scattering by lattice vibrations and impurities in the passive region i.e., $\epsilon < \epsilon_0$; ϵ_0 is the energy of an optical phonon; and τ^+ is the characteristic time for the emission of an optical phonon in the active region, $\epsilon > \epsilon_0$ $(\tau_1^+ = \tau_2^+ = \tau^+)$]. These conditions can be satisfied at $kT \ll \epsilon_0$. We fix the magnetic field at $H = 23$ kOe. If E is such that $p_{1_H} > p_{1_0} = \sqrt{2m_1\epsilon_0}$ but $p_{2_H} < p_{2_0}/2$, then the main trajectory (which passes through the point $p_x = p_y = 0$) of the heavy holes is broken (it goes outside the passive region, and there is no "spindle,"[2,3] whereas the main trajectory of the light holes is closed (it lies entirely in the passive region; see the solid curves in Fig. 1). If $\omega_{c_1}\tau^+ \ll 1$ and $\omega_{c_1}\tau_1^- \gg 1$, the heavy holes are not scattered as they repeatedly orbit along the closed trajectories in the passive region, and they do not penetrate deeply into the active region; after the emission of a phonon, they return to bands 1 and 2, to the point $p = 0$ (so that $p_{0_x} = p_{0_z} = 0$) (Ref. 1). The light holes spend a time τ_2^- on closed trajectories of the passive region and are then scattered, primarily into the heavy-hole band. The scattering of heavy holes is more frequent than that of light holes (since $\omega_{c_1}^{-1} \ll \tau_2^-$), so that the difference between the actual number of light holes and the equilibrium number increases in proportion to $\omega_{c_1}\tau_2^-$. If $p_{1_0}/2 < p_{1_H} < p_{1_0}$ and $p_{2_H} < p_{2_0}/2$, the situation is less favorable with regard to the change in population, since a region in which holes accumulate forms in the heavy-hole band. This region is spindle-shaped in momentum space. At $p_{2_H} > p_{2_0}/2$, the main trajectory of the light holes is broken, and the holes may be scattered more frequently. The change in the population of band 2 should decrease.

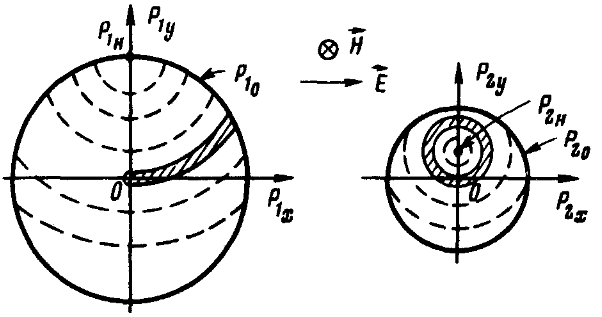

FIG. 1. Trajectories of heavy and light holes in the $p_z = 0$ plane in momentum space.

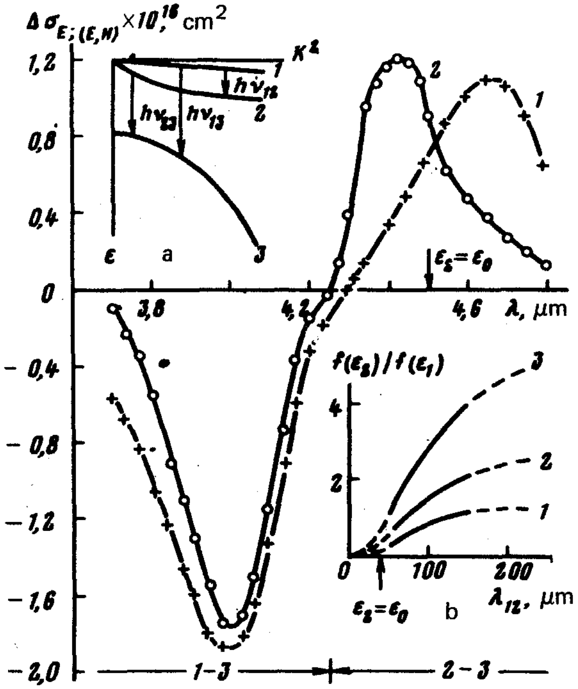

FIG. 2. Spectral dependence of the change in the absorption cross section of p-Ge for infrared light for the transitions 1–3 and 2–3. $T_0 = 18$ K, $E = 4$ kV/cm. 1–$H = 0$; 2–$H = 23$ kOe. $p = 2.3 \times 10^{14}$ cm^{-3}. Inset a shows the structure of the p-Ge valence band and the possible optical transitions. Inset b shows the wavelength dependence of the ratio of the distribution functions of the light holes, $f(\epsilon_2)$, to that of the heavy holes, $f(\epsilon_1)$, for the 1–2 transitions. $T_0 = 18$ K, $H = 23$ kOe. 1–$E = 2.14$; 2–4; 3–5.3 kV/cm.

In an attempt to observe the transfer of holes from band 1 to band 2 and to observe a population inversion, we studied the change in the infrared absorption in p-Ge in strong fields $\mathbf{E} \perp \mathbf{H}$ at $T_0 = 18$ K. The impurity concentration was $N \sim 10^{15}$ cm^{-3}, and the hole concentration at $T > 40$ K was $p = 2.3 \times 10^{14}$ cm^{-3}. At lower temperatures, the impurities are "frozen," but in the fields used in the present experiments all the impurities were ionized because of breakdown. The shape of the sample was such that the Hall field was largely short-circuited, so that the total field was roughly equal to the applied field. Figure 2 shows the change in the absorption cross section, $\Delta\sigma$, in a field $E = 4$ kV/cm ($H = 0, H = 23$ kOe); also shown here are the spectral intervals for the 1–3 and 2–3 transitions, which correspond to transitions from bands 1 and 2 to band 3, the latter being split by the spin-orbit interaction. The field $E = 4$ kV/cm corresponds approximately to the situation in Fig. 1. We first note that there is an increase in the number of light holes at low energies, $\epsilon_2 < \epsilon_0$ ($\lambda = 4.25$-4.505 μm) when the magnetic field is applied. That the observed effect is not simply a cooling in the magnetic field is confirmed by the fact that in weaker electric fields (with less heating) and with $H = 0$ the absorption modulation spectrum does not agree with that observed in a strong magnetic field in terms of either the spectral position of the extremum or its magnitude. The heavy-hole energy distribution at low energies ϵ_1 (the spectral interval for the transitions 1–3 in Fig. 2 corresponds to ϵ_1 near $\sim 10^{-2}$ eV) does not undergo any important changes in a magnetic field. The extremum of $\Delta\sigma_E$ at $H=0$ results from a singularity of the reduced state density.[5] In the magnetic fields used, we have $\omega_{c_1} = 1.15 \times 10^{12}$ rad/s, $\omega_{c_2} = 9.7 \times 10^{12}$ rad/s, $\omega_{c_1}\bar{\tau}_1 = 22$, and $\omega_{c_2}\bar{\tau}_2 = 145$; in other words, the heavy holes penetrate quite deeply into the active region (by about $1.3\, p_0$, which corresponds to $\epsilon_1 \sim 1.8\, \epsilon_0$), and after the phonon emission they are scattered uniformly over the passive region of both bands 1 and 2. Consequently, in addition to the motion along the main trajectories, some of the holes move along the trajectories shown by the dashed curves in Fig. 1, and this effect can reduce the change in the population of band 2. Study of $\Delta\sigma_{E,H}$ (λ) over the range of fields E from 1 to 6.4 kV/cm shows that the number of light holes in the passive region increases smoothly with increasing E, without undergoing any sharp changes at $E = 2.25$ kV/cm ($p_{1_H} \approx p_{1_0}/2$, $E = 4.5$ kV/cm ($p_{1_H} \approx p_{1_0}$), or $E = 5.57$ kV/cm ($p_{2_H} \approx p_{2_0}/2$). The observed effect can be attributed to the formation of spindle-shaped regions in which heavy and light holes accumulate; to a decrease in their phase volume with increasing E (Ref. 3), which leads to a carrier redistribution in momentum space; and to a penetration of holes deep into the active region, which leads to a blurring of both the main trajectory and the spindle. Let us use the experimental data to estimate the hole population inversion at the energies for the 1–2 transitions, which correspond to infrared wavelengths of 50-150 μm. For this purpose we must determine the distribution function of the heavy holes in the range $\epsilon_1 \sim (3\text{-}1) \times 10^{-3}$ eV and that for light holes in the range $\epsilon_2 \sim (2.5\text{-}1) \times 10^{-2}$ eV (Refs. 6 and 7). The heavy-hole distribution function $f(\epsilon_1)$ was found in the range $\epsilon_1 \sim (5.5\text{-}1) \times 10^{-2}$ eV in Ref. 5 at $T_0 = 82$ K. It can be found for smaller values of ϵ_1 through an extrapolation. In the determination of the distribution at $T_0 = 18$ K, it was assumed that the size and shape of the distribution function were independent of T_0 in sufficiently strong fields E and that the correction for the magnetic field could be incorporated easily, since $\sigma_{13} \sim f(\epsilon_1)$. It is a complicated matter to determine the distribution function of the light holes from the light absorption over a broad interval of ϵ_2, for the reasons discussed in Ref. 5. Nevertheless, in the range of ϵ_2 of interest here, the calculations yield approximately $\sigma_{23} \sim f(\epsilon_2)$, and the distribution function can be found by the method described in Ref. 5 (for example). The results of such calculations are shown in Fig. 2. The increase in the inversion with increasing E results from both an increase in $f(\epsilon_2)$ and a decrease in $g(\epsilon_1)$.

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