Elastic ed scattering at small Q^2 and charge form factor of the neutron

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A relativistic representation of the charge form factor of the deuteron is used to determine the charge form factor of the neutron, $G_{En}(Q^2)$, at small Q^2 and to determine the slope of this form factor at the origin, $G'_{En}(0)$, from experimental data on elastic ed scattering. For the first time, a slope in approximate agreement with the experimental value, $G'_{En}(0) = 0.0182$ F^2, has been found by an approach which does not involve the conventional method of deuteron wave functions.

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The prolonged scattering of thermal neutrons by the electron shells of various atoms (the inert gases, W, Pb, and Bi) leads to a stable, positive value of the slope[1)] of the charge form factor $G_{En}(Q^2)$ at the point $Q^2 = 0$ (Ref. 1),

$$G'_{En}(0) = 0.0199 \pm 0.0003\ F^2. \tag{1}$$

The rms radius of the charge distribution in the neutron, $\langle r_n^2 \rangle = -6G'_{En}(0)$, is thus nonzero and negative. This means that the neutron is a carrier of an extremely unusual charge structure: The neutron itself has no net charge, there is a charge distribution (of a sign and magnitude unusual in hadron physics), and there is no simple relationship between the charge and magnetic structures. These features sharply distinguish the neutron from the proton and other charged baryons. The many published attempts to interpret this charge distribution on the basis of quark models are discussed in Ref. 2 (see also Ref. 3). In the present letter we wish to discuss another fundamental aspect of the problem of calculating the radius $\langle r_n^2 \rangle$: Since the most reliable source of information on the magnitude of $G_{En}(Q^2)$ over a broad range of Q^2 is elastic electron-deutron scattering, it is first necessary to show that the charge form factor found for the neutron from *ed* scattering with the help of some model for the deuteron reproduces the experimental value of the slope at the origin, (1). For a long time this problem remained unresolved. Not until 1973 was progress made in its solution, by Berard *et al.*,[4] after (first) the refinement of earlier measurements[5] of the cross section for elastic *ed* scattering at small Q^2 ($<1\ F^{-2}$) and (second) the appearance of the pioneering paper by Casper and Gross.[6] The latter emphasized the purely relativistic nature (even in the limit $Q^2 \to 0$) of the problem of calculating $G'_{En}(0)$ from *ed* scattering and were the first to develop a workable relativistic correction to the nonrelativistic formalism. The calculations by this combined approach in Refs. 4–6 demonstrate that the deuteron wave functions from the Feshbach–Lomon model are clearly preferable. All the other basic and realistic deuteron wave functions lead to unacceptable values for the slope (values which are too large or essentially zero).[7]

In the present letter we will calculate the charge form factor of the neutron and its slope from the results of the most accurate measurements[4] of the elastic *ed* cross section at small Q^2 and from a compact relativistic integral representation[8] of the charge form factor of the *S*-wave deuteron, $G_c^d(Q^2)$ (without any reference to a nonrelativistic formalism). This representation is[1])

$$G_c^d(Q^2) = [G_{Ep}(Q^2) + G_{En}(Q^2)]\, D_c(Q^2), \tag{2}$$

$$D_c(Q^2) = \frac{\Gamma}{[1+Q^2/4M^2]^{1/2}} \frac{1}{[B'(M_d^2)]^2} \int_{4M^2}^{\infty} \frac{ds\, \Delta B(s)}{s - M_d^2}$$

$$\times \int_{s_1(s,Q^2)}^{s_2(s,Q^2)} \frac{ds'\, \Delta B(s')}{s' - M_d^2} \frac{(s+s'+Q^2)^2}{[(s-4M^2)(s'-4M^2)]^{1/2}} \frac{Q^2}{[\lambda(s,s',Q^2)]^{3/2}}, \tag{3}$$

$$B(s) = \left(1 - \frac{M_d^2 - 4M^2}{s - 4M^2}\right) \exp\left\{\frac{1}{\pi} \int_{4M^2}^{\infty} \frac{du\, \delta(u)}{s-u}\right\},$$

$$s_{2,1} = 2M^2 + 1/2\, M^2(2M^2 + Q^2)(s - 2M^2) \pm 1/2\, M^2 [Q^2(Q^2 + 4M^2)s(s-4M^2)]^{1/2},$$

$$\lambda = s^2 + s'^2 + Q^4 + 2(sQ^2 + s'Q^2 - ss').$$

In (3), Γ is a normalization constant,[2]) $\Delta B(s) = B(s+i\epsilon) = B(s-i\epsilon)$, M and M_d are the masses of the nucleon and the deuteron, respectively, and $\delta(s)$ is the triplet phase shift for *np* scattering. The results calculated for the structure function $D_c(Q^2)$ with the experimental phase shifts are given in Ref. 8. In extracting $G_{en}(Q^2)$ from *ed* scattering it is also necessary to consider the contribution of the meson exchange currents (G_c^{MEC}) to G_c^d and the contribution of the quadrupole form factor of the deuteron (G_Q^d) to the experimental cross section for *ed* scattering.[3]) Simple expressions for these contributions for small values of Q^2 are given in Ref. 7. Using D_c, and taking into account the contributions G_c^{MEC} and G_Q^d, we can calculate the charge form factor of the neuteron from

$$G_{En} = G_{Ep}\left\{\frac{1}{D_c}\left[R - \frac{1}{G_{Ep}} 2G_c^{MEC} D_c - \frac{1}{G_{Ep}^2}[(G_c^{MEC})^2 + (G_Q^d)^2]\right]^{1/2} - 1\right\}. \tag{4}$$

Here R is the ratio of the cross sections for elastic *ed* and *ep* scattering, which was calculated in Refs. 4 and 5; $G_c^{MEC} = -2.5 \cdot 10^{-3} \cdot Q^2(\Phi^2)$; and $(G_Q^d)^2 = 4.5 \cdot 10^{-3} \cdot$

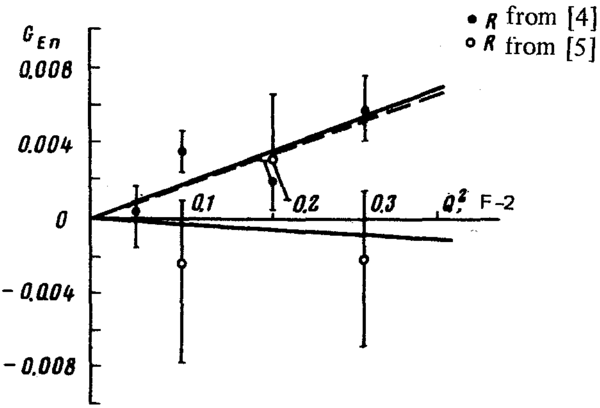

FIG.1. Values of $G_{En}(Q^2)$ calculated from Eq. (4) and the results of two experiments, Refs. 4 and 5. The lines show the slopes of the two versions of the form factors in the linear fit $G_{En}(Q^2) = bQ^2$, with the respective values $b = 0.0197$ F^2 and $b \approx 0$.

$\cdot\, Q^4 (\Phi^{-4})$. The results calculated for $G_{En}(Q^2)$ are shown in Fig. 1. The earlier measurements of R in Ref. 5, which were less accurate than those of Ref. 4, led to an oscillatory behavior of G_{En} and to an essentially zero slope for this form factor. The refinement of R by Berard *et al.*,[4] "raises" G_{En} substantially and makes it stably positive at all Q^2. To find $G'_{En}(0)$ in accordance with the procedure of Ref. 4, we write $G_{En}(Q^2)$ as a second-degree polynomial, $G_{En}(Q^2) = bQ^2 + cQ^4$, whose coefficients are determined by a fit with the values calculated from Eq. (4). For the value of G_{En} found with the help of the value of R from Ref. 4, this procedure leads to the slope $G'_{En}(0) = 0.0182$ F^2, which is only 8% smaller than the observed slope in (1). By way of comparison, we note that the various versions of the Feshbach-Lomon deuteron wave functions lead[7] to slopes ranging from 0.0215 F^2 to 0.0257 F^2. The relativistic representation in (2) and (3) thus leads to a charge form factor for the neutron which reproduces the experimental value of the slope, (1), quite accurately. We can also see from Fig. 1 just how sensitive the charge form factor of the neutron is to the refinement of the experimental values of R. Since the cross section for elastic *ed* scattering has been measured in the interval $0 < Q^2 < 0.1$ F^{-2} in only one experiment,[4] and for only the single point $Q^2 = 0.05$ F^{-2}, it would be extremely useful to have some new and more detailed measurements in this interval (with an accuracy at least matching that of Ref. 4).

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[1)] Everywhere below, $Q^2 = -q_\mu q^\mu > 0$.
[2)] Equation (2) differs from the equation in Ref. 8 in that it omits the contribution of the relativ-

istic spin rotation [it is necessary to set $\chi = 0$ in Eq. (3) in Ref. 8]. In the S-wave deuteron this effect leads to the appearance in G_c^d of a new term ($\sim \sin\chi G_{MN}^S$), which is not found in any other approach. At small values of Q^2, the resultant effect of the relativistic spin rotation is a small quantity of order $(v/c)^4$, which may be discarded in a calculation of $G'_{En}(0)$.

3) In Ref. 8, Γ was calculated from the condition $G_c^d(0) = 1$. This redefinition of the S-wave charge form factor of the deuteron effectively incorporates the D-state contributions and the contributions of the left-hand cuts in $G_c^d(Q^2)$; these effects are negligible at small values of Q^2 and unimportant in a calculation of $G_{En}(Q^2)$. This circumstance was mentioned in a slightly different form in Ref. 9.

4) The baryon degrees of freedom (the $\Delta\Delta$ configuration in d) are not important in a calculation of $G_{En}(Q^2)$ at small values of Q^2 (Ref. 7).

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